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Training in High School Mathematics Essential for Life*

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THE magazine *Time* for January 5, 1942 devotes one page to a discussion of the revival of interest in mathematics. The article mentions improvements in mathematics courses over the country and calls attention to the demands of army and navy officers for more and better mathematics. Two sentences on the page attracted my attention: "Mathematicians have long been haunted by a paradox. Although most U. S. citizens profess to dread the study of mathematics, they are suckers for mathematical puzzles, made a best-seller of Lancelot Hogben's *Mathematics for the Million*. The mathematicians' conclusion—the trouble is not with mathematics, but with the way it is taught."

It is not my intention to summarize articles and books dealing with *Mathematics of Aviation, Mathematics for Defense. Services of Mathematics to Nurses, Pharmaceutical Mathematics, Mathematics of Investment, Mathematics and Field Artillery, Mathematics for Chemistry Students, Biomathematics*, and so on. There are certain facts, skills, and concepts which are essential in specialized fields of endeavor. Furthermore, it is quite evident that the number of such fields is increasing rapidly, and will continue to increase. It was Adolph Quetelet, the great Belgian astronomer who in the nineteenth century said, "The more advanced the sciences have become, the more they have tended to enter the domains of mathematics."

I am profoundly interested in "the way mathematics is taught" in order that the individual during his school life and in later life may profit from the subject even though he may not be in a specialized field which requires it. This discussion must necessarily border upon the dangerous field of Formal Discipline and Transfer of Training. The psychologist's acceptance of our statements depends quite largely upon our choice of words. Several years ago I used the well-known "right hand rule" of physics as an illustration of visualization. I was informed that certain psychologists objected to the term "Visualization." Thereupon, I consulted an outstanding psychologist, stated the rule: "If you think of your right hand as grasping a wire through which an electric current is flowing with the thumb pointing in the direction of the current, the fingers will point around the wire in the same direction as the magnetic field," and asked him what he would call it if it is not visualization,

* Paper read at the Annual Meeting of the National Council of Teachers of Mathematics at San Francisco, February 21, 1942.

since you do not actually grasp the wire. Apparently he was nonplussed for a moment, but soon arose to the occasion and replied: "That would be 'using a verbal rule as a stimulus to a process carried on in the imagination'." It is evident that our choice of words is important.

As a general statement of the objectives in the teaching of mathematics, there is probably none better than that of the National Committee in *The Reorganization of Mathematics in Secondary Schools:* "The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make these powers effective in the life of the individual."

After re-reading Vevia Blair's article *The Present Status of Disciplinary Values in Education* which appeared in the *National Committee Report*, and the article by Dr. Pedro Orata in THE MATHEMATICS TEACHER in 1935, I am still firm in the belief that certain ideals, appreciations, attitudes, and abilities can be developed through the teaching of mathematics.

Twenty-one years ago John A. Marsh wrote an article which appeared in the magazine *Educational Administration and Supervision*. This article deals with the scholastic success of students who had elected mathematics as compared with those who had not elected mathematics in the Boston English High School for boys. From the records of approximately fifteen hundred boys who were graduated between 1914–1919, Mr. Marsh selected 115 boys who had elected one year and one year only of what we now call general mathematics in the ninth grade and matched these with 115 other boys who had not taken any high school mathematics. Each pair of boys had pre-high school work in the same elementary school of Boston, with approximately the same Z-mark. The Z-mark was obtained by a weighted average of all grades for subjects taken in the elementary schools.

Table I simply shows that Mr. Marsh did a good job in pairing the 115 boys. You may observe that in all subjects combined, the boys who did elect mathematics earned almost exactly the same number of A's and B's as the boys who did not elect mathematics. This is as it should be if they were originally paired according to scholastic success. The evidence in Table II indicates that something must have happened to the boys who studied mathematics during their first year. Please note that in every subject with the exception of civics, the boys who had studied mathematics had a larger per cent of A's and B's than did the boys who had not studied mathematics. We do not dare to use the word "transfer." The simple fact is that for some reason the group of boys who had taken mathematics earned more A's and B's. Comparing each of the 115 boys who elected mathematics with his teammate who did not elect mathematics 58% excelled, 30% fell below, and 12% showed no difference.

In the case of English, there is a superiority of 26%. We have no way of knowing definitely why these boys did superior work. Possibly, in their mathematics classes, they learned to appreciate the value of orderly arrangement of material. Note the wording: "learned to appreciate the value of," not "learned the habit of." Paragraphing in English very definitely demands an orderly arrangement of material.

If the student of mathematics is properly trained, he should recognize the value of careful reading. Careful reading is certainly an asset in English. Concise, unambiguous statements are highly essential in all phases of mathematics. One objective in English is conciseness. The teacher of mathematics finds from experience that the pupil who is neat and painstaking in his work has fewer mathematical errors than the one whose work is so

poorly arranged and so poorly written that he makes mistakes in reading the symbols. Experiments in English have shown that the student who has a typewritten theme will receive a higher grade than the student with the same paper written in longhand. In other words, neatness and care in writing are valuable in English, and appreciation of the value of neatness in mathematics should help pupils to be more neat in English. In the mathematics class, the pupil has something definite to say and the teacher is in a position to demand the correct, concise and unambiguous statement of this fact. Most pupils in writing an English theme spend more time in deciding upon something to say than in the actual writing of the theme. In the case of mathematics, the pupil has something to say and the training he gets in stating his argument or proof in good English in many cases is more valuable than the training he gets in expressing himself in an English class.

In French and Spanish, it is rather difficult to even guess at the cause of a superiority of fifteen per cent in favor of the boys who had elected mathematics. Mathematics is definitely a study of relationships. Functionality has been emphasized for many years. The concept of functionality as shown in geometric relationship, in algebraic equations and graphs furnishes the backbone of the mathematics course. Possibly this appreciation of relationships may carry over to a foreign language in relationships which determine the verb forms, and the endings of certain nouns and adjectives.

In history, the table shows a superiority of thirty-two per cent in favor of the mathematics students. We have never suspected that there is much relationship between success in mathematics and success in history. Too often history is taught as a memory subject. None of us advocates memorizing as an essential part of mathematics. However, one important phase of history is the outlining of the subject. Outlining means logical organization of material, perhaps logical organization of *cause* and *effect*. This phase of history is definitely emphasized in mathematics.

The next subjects in the table are physics and chemistry. We are not surprised that in their case the superiority is fifty per cent in favor of the mathematics student. This, of course, is somewhat due to the occurrence of identical elements. Anybody would admit that training in mathematics formulas and in arithmetic computation would help students in their work in formulas in the sciences. No other high school subject furnishes an opportunity to impress upon the pupils the seriousness of inaccuracy as does mathematics. In the case of English, a pupil may write an excellent theme and yet have a misspelled word. A *lawyer* in his plea before the jury may move the members to tears and yet mispronounce a word. A *journalist* can write an editorial that will sway his reader to his point of view and yet have a grammatical error in a sentence or two. A *preacher* may convert many souls and yet mispronounce words in his sermon. But a mathematician or an engineer who makes the slightest computational error will invalidate all of his conclusions. Appreciation of accuracy is a logical outcome from the proper teaching of mathematics. Accuracy is, of course, a primary essential in physics and chemistry. The habit of accuracy will not necessarily "carry over," but an appreciation of its value can be developed.

The laboratory work in physics and chemistry requires keen and accurate observation. Again this is an essential for success in mathematics and a teacher may well have this as one of the objectives in teaching her subject. If then, procedure in geometry is quite closely related to cause and effect, as it occurs in physics and chemistry experiments again and again, an appreciation of the value of carefully studying cause and effect in mathematics may well carry over to the appreciation of the careful observation

of the phenomena in science experiments.

In bookkeeping as in the sciences, we are not surprised to find a superiority as large as thirty-nine per cent in favor of those who had mathematics. There are many identical elements in work in mathematics and in work in bookkeeping and, of course, the appreciation of the value of accuracy is again paramount.

Beginning with commercial geography, you note that some items are enclosed in parentheses. The author felt that the numbers were so small in these cases that a comparison of percentages is not so very significant. The superiority in commercial geography, commercial law and drawing; and the inferiority in civics are in each case computed from a small number of cases and, therefore, do not contribute much to the study.

In the case of stenography the superiority is much more than in typewriting. Stenography requires careful observation and study of form. The slightest change in a character completely changes the meaning. Students of geometry have learned to observe very carefully differences in geometric forms. This one item of experience may readily account for the fact that the boys who had geometry did so much better than did the boys who had not had geometry. Why were the boys who had mathematics superior in typing? A statement made by a banker who was addressing a group of high school teachers may suggest the answer. His first statement was that their banking firm would not think of hiring a boy or girl for even a minor position without their having a high school education. This, of course, pleased the teachers and they showed their appreciation of the statement by cheering the speaker. Then he hastened to say: "Please don't misunderstand me. It is not because of anything that you have taught these boys and girls, but merely because it gives us evidence that they have the proper quality of stick-to-it-iveness, as they have stayed by their high school for four years or more when you have given them so much material that is distasteful drudgery." Possibly our work in mathematics has produced in the students a willingness to continue through tedious and tiresome drills for the sake of acquiring efficiency. This may be an explanation of the willingness of our mathematics boy to go through the same sort of tedious drill in practice in typewriting in order that he may be able to become efficient in that work.

In the *Fifteenth Yearbook—The Place of Mathematics in Secondary Education*—you may find these sentences: "Many fail to accomplish meritorious work within their possibilities solely because they do not have the quality commonly described as 'stick-to-it-iveness.' . . . Voluntary acceptance of discipline is needed in a democracy, and it is a duty of the schools to develop it as a product of the life of the school as a whole."

In the *National Committe Report* previously mentioned, Clifford Brewster Upton referred to the same trait when he mentioned the ability and willingness to read an income tax blank, an accident insurance policy, or a deed. Upton listed the following essentials:

1. The development of the patience and willingness to go through a mass of details, keeping each item in mind.
2. The ability to do close and detailed thinking.
3. The ability to give sustained attention.
4. The ability to weight the significance of each word in a sentence.
5. The ability to know whether a sentence has meaning or not.
6. The ability to think rigorously and soundly rather than carelessly.
7. The inquiring or questioning attitude of mind.
8. The ability to subject a statement to a severe test of its truth or validity.
9. The ability to discover whether sufficient data have been given.
10. The ability to analyze.
11. The ability to draw conclusions.

We do not dare to hope that everyone who studies mathematics will acquire these abilities. However, we can at least endeavor to develop an appreciation of the value of such abilities.

Within recent years, two graduate students under my supervision have completed studies in the schools of Lincoln, Nebraska, similar to that of Mr. Marsh. Their findings were in absolute accord with his. A summary of one of these studies follows:

Miss Mildred Waide, a mathematics teacher in the Lincoln schools, examined the records of approximately *4500* pupils who completed one of two junior high schools over a *seven-year period*. From this number she found *125* pupils who did not elect to take mathematics in the ninth grade of the junior high school, and who did not take any senior high school mathematics. The ninth grade course in mathematics is either formal algebra or general mathematics. Miss Waide then computed the average grade in *English*, the *social studies* and *science* for each of these 125 pupils. Then she matched each of these pupils with one from the same school who had elected ninth grade mathematics and whose average grade in the subjects just mentioned was the same. Having these 125 pairs of pupils who completed the ninth grade, Miss Waide then examined their records in English, the social studies and the sciences through the senior high school. In English, *54.4* per cent of those who had elected mathematics had higher grades than their teammates who had not elected mathematics. Forty per cent of those who did not take mathematics had higher grades than their teammates who had taken mathematics. The remaining *5.6* per cent had identical grades. Miss Waide also applied the statistical technique for the significance of difference of means. The ratio of the difference of the means of the two groups to the standard deviation of the difference of the means was *2.90*. Statistically interpreted, this means that there are *99.8* chances in one hundred that if the same investigation were repeated under similar circumstances, the students who had elected mathematics in the ninth grade would have a higher average in English than those who did not.

The same tabulation was made for grades in science with results much more definitely in favor of the mathematics students. This, of course, is not surprising. *58.8* per cent of those who had had mathematics excelled their teammates who had not elected mathematics, whereas *27.78* per cent of those who did not take mathematics were superior to their teammates. The same statistical technique showed the ratio between the difference of the means and the sigma of the difference of the means to be *4.78*. A ratio of three is considered completely significant. Anything larger than that indicates a practical certainty that with other investigations under like conditions, those who had elected mathematics would have the higher average.

The most surprising results were in the case of social studies. Here *63.2* per cent of those who elected mathematics had higher grades than their teammates, whereas only *30.4* per cent of those who had not elected mathematics had grades above those of their teammates. The ratio in this case was *4.61* which again indicates practical certainty that the pupils who elected mathematics would have the higher average.

Note that these pupils were originally paired on exactly the same subjects that were used to compare their grades in the Senior High School.

Lancelot Hogben, author of *Mathematics for the Million*, in his later book, *Dangerous Thoughts*, makes the statement in referring to mathematics: "No other subject offers the teacher such great opportunity for wrecking the intellectual self-confidence of the pupil." He may be right. However, I feel quite confident that no other subject offers the teacher such a wonderful opportunity to develop self-

TABLE I

Showing the Relative Standing in the First Year of High School of Pupils Who Did Study Mathematics in That Year and Those Who Did Not

Subjects	Group	No. of cases	Distribution of grades by per cents[1]					
			A	B	C	D	A & B	C & D
English	Math.	115	6	22	69	3	28	72
	Non-Math.	115	7	31	58	3	38	61
French and Spanish	Math.	105	10	37	47	6	47	53
	Non-Math.	104	10	27	59	5	37	64
English, French, and Spanish	Math.	220	8	29	58	5	37	63
	Non-Math.	219	8	29	59	4	37	63
Bookkeeping	Math.	78	9	33	56	2	42	58
	Non-Math.	115	2	32	64	2	35	66
History, Science, and Drawing	Math.	47	4	35	56	4	39	60
	Non-Math.	123	8	37	53	2	45	55
All Studies except Mathematics	Math.	555	8	31	58	4	39	62
	Non-Math.	676	7	32	59	3	39	62
All Studies	Math.	676	8	32	58	3	40	61
	Non-Math.	676	7	32	59	3	39	62

[1] Grades A, B, and C are passing grades. Grade D indicates failure.

TABLE II

Showing the Relative Standing in the Second and Third Year of High School of Pupils Who Studied Mathematics in the First Year and of Those Who Did Not Study Mathematics in the First Year of High School

Subject	Group	No. of cases	Distribution of grades by per cents					Superiority of mathematical group in per cents of A and B grades
			A	B	C	D	A & B	
English II	Math.	115	3	26	64	6	29	26
	Non-Math.	115	3	20	64	12	23	
English III	Math.	115	3	21	66	10	24	26
	Non-Math.	115	2	17	69	13	19	
French and Spanish	Math.	171	2	21	66	11	23	15
	Non-Math.	171	3	17	67	13	20	
History	Math.	53	6	23	66	6	29	32
	Non-Math.	64	6	16	75	3	22	
Physics and Chemistry	Math.	78	9	27	53	12	36	50
	Non-Math.	60	7	17	73	3	24	
Bookkeeping II	Math.	108	7	25	66	2	32	39
	Non-Math.	112	2	21	68	9	23	
Bookkeeping III	Math.	80	10	26	57	6	36	29
	Non-Math.	67	7	21	66	6	28	
Commercial Geography	Math.	30	0	13	87	0	13	(225)
	Non-Math.	44	2	2	84	11	4	
Commercial Law	Math.	19	5	16	79	0	21	(24)
	Non-Math.	35	6	11	74	9	17	
Stenography II	Math.	111	9	32	53	6	41	41
	Non-Math.	104	8	21	61	10	29	
Stenography III	Math.	85	8	35	53	4	43	30
	Non-Math.	87	7	26	58	9	33	
Typewriting	Math.	100	17	27	54	2	44	19
	Non-Math.	102	8	29	57	6	37	
Drawing	Math.	15	0	40	53	7	40	(8)
	Non-Math.	38	3	34	63	0	37	
Civics	Math.	20	0	25	70	5	25	(−32)
	Non-Math.	9	0	33	56	11	33	
All Studies	Math.	1100	7	25	62	6	32	28
	Non-Math.	1123	5	20	66	9	25	

confidence. I recently used two texts that had many errors in the answers. My students are required to make daily reports on their preparation. At the beginning of the semester, they would report that they could not get certain problems. A little later they would venture to suggest that possibly the answer in the book was wrong. By the end of the semester frequent reports would state "The answer in the book is wrong."

We do not dare to claim that the habit of concentration can be developed in mathematics, but we can state with some assurance that no student can solve a geometry original unless he can concentrate to the extent of carrying in his mind the given facts and the required facts, being able to shut out all outside interference until he can establish a relationship between these known and required facts. This requires a certain amount of concentration over a period of several minutes. Compare this with the learning of a vocabulary in a foreign language. Our student can learn the first word, then listen to the radio for a while, learn another word, and enter into a conversation with the family, learn another word, and call a pal over the telephone. Eventually after wasting considerable time, he will have his vocabulary memorized without discovering the value of concentration. In the case of the geometry exercise such a procedure is an absolute impossibility. The teacher of geometry has an opportunity to convince the pupil of the necessity of concentration and to lead him to admire and desire that ability. A personal experience illustrates the point. If I attempt to read a book on history, philosophy, sociology, or even a novel in a room where others are talking, I find it difficult to concentrate on the subject before me. However, I have no difficulty whatever under the same conditions in solving a problem in algebra or trigonometry if I can use pencil and paper. The moving pencil point as the solution evolves focuses my attention and carries me from one step to the next. Result? I recognize the necessity and the value of concentration. I know how my attention is focused in the mathematics problem. I therefore attempt a similar procedure, and move my pencil along the printed page, frequently marking important phrases as I read a book in history or philosophy.

Does the study of geometry make us more logical? Not necessarily. E. L. Thorndike in his *Principles of Teaching* written several years ago made the following statement: "The study of geometry may lead a pupil to be more logical in all respects, for one element of being logical in all respects is to realize that facts can be absolutely proven, and to admire and desire this certain and unquestionable sort of demonstration." Parker's text on *Methods of Teaching in High Schools* uses mathematics for his illustrations in his chapter on "Reflective Thinking."

Over twenty years ago, Rugg and Clark, under the title "Scientific Method in the Reconstruction of Ninth Grade Mathematics" made the following statement: "The most important outcomes in the study of algebra are thought outcomes, that is, the ability to think logically, to use principles, to see relationships, to pick out essentials, to analyze, and to organize." Everett expresses a similar belief in his *Fundamental Skills of Algebra* when he says: "The child needs algebra for what it will enable him to do, but infinitely more does he need it for the orderly way in which, in the midst of constant change, it will enable him to think."

Several recent writers have emphasized the relationship between "straight thinking" and the study of mathematics. H. C. Christofferson in THE MATHEMATICS TEACHER for April, 1938, in an article entitled "Geometry—A Way of Thinking" brings out the importance of a procedure or a method of attack which may be applied to other fields. *The Thirteenth Yearbook on The Nature of Proof* by Harold

Fawcett illustrates an elaborate application of mathematical thinking to life problems. A recent dissertation by Gilbert Ulmer at the University of Kansas entitled *An Experiment in Teaching Geometry to Cultivate Reflective Thinking* presents evidence that it is possible for teachers to improve the ability of pupils in reflective thinking.

The title of this paper is "Training in High School Mathematics Essential for Life." Life situations require precisely the ideals, attitudes, appreciations, and abilities just mentioned. The study of mathematics does not necessarily produce any of these, but they are valid aims in the teaching of mathematics. Furthermore, by the very nature of the subject mathematics presents opportunities for emphasizing these desirable outcomes as does no other subject.

Accuracy, precision, orderliness, clarity, concise English, thoroughness, neatness are outstanding ideals essential for success in any mathematics class. The individual who has learned to admire and desire these qualities is fortunate.

An appreciation of the necessity for organization, classification, analysis, generalization, logical thinking, suspended judgment, should be developed in every mathematics class.

The ability to use a symbolic language in order to condense whole paragraphs into a few symbols is characteristic of those trained in mathematics, and is essential in many fields of reasoning.

Persistence in wading through cumbersome and tiresome details for the sake of the thrill that results from a task well done is a trait that results from many problems in mathematics.

Mental self-reliance and ability to think while speaking before an audience rather than depending upon ability to recall facts previously memorized are the logical results of the orthodox methods of conducting classes in mathematics.

The recognition of functional relationships, irrelevant data, failure to establish hypotheses from which conclusions are drawn, cause and effect, misrepresentation by means of graphs, errors in quoting authorities, all of which are illustrated in mathematics, would certainly change many jury decisions, prevent the acceptance of the majority of political speeches, and ruin the sale of patent medicines.

Junior Colleges

IN SPITE of unusual losses due to wartime closing of several institutions, the junior colleges of the country have held their own in numbers and have increased their services to the youth of the country, according to data appearing in the *Junior College Directory 1943*, just published by the American Association of Junior Colleges.

The new publication, edited by Walter C. Eells, Executive Secretary of the Association, shows 624 junior colleges in 45 states in January 1943, as compared with 627 institutions at the same date in 1942. While 30 institutions have closed on account of decreased enrollments or because the government has commandeered their plants for the duration, the loss has been compensated for by 27 junior colleges newly organized or listed for the first time, with enrollments much greater than in the institutions which have closed.

Total junior college enrollments reported are in excess of 314,000 students, a growth of more than 17 per cent over the previous year. There has been a slight decline of 9,000 regular students, but a marked increase of 56,000 in the number of special students. In many cases these special students are adults employed in war industries or taking special courses to prepare them for such employment.

"While the junior colleges of the country will be hard hit by the lowering of the draft age, and some others may be forced to close for the duration," said Secretary Eells, "most of them are making emergency adaptations of their curricula to meet local needs and are doing their full part in many ways to contribute substantially to the success of the total war effort."

The largest number of junior colleges are found in California, with 69 such institutions having a total enrollment of 144,000 students. Texas is second with 42 junior colleges and 18,000 students.

Guidance for Girls in Mathematics

By HARRIETTE BURR
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THE CHALLENGE has been given to womankind: A WOMAN TRAINED TO DO THE WORK OF EVERY MAN NEEDED ON THE BATTLEFRONT. Herein is one of the greatest contributions of teachers to the manpower problem in this war effort: to see that each girl, as well as her family, hears this challenge; and to so guide a girl in her program of studies that she is becoming a specialist in some one field of work in order that she will be competent to assume her duties in the war effort.

In this manpower crisis the need for women workers is acute; but the shortage in technical and scientific fields is crucial. New and broader vocational opportunities are opening for women in engineering, science, and craft work. But there is more than opportunity involved; today it is an obligation for those with talent and interests in these fields to enter them.

For the individual teacher counselling the individual pupil, there is often a sense of conflicting responsibilities in balancing the destiny of the pupil against the need of society. To secure this balance, it is essential that the teacher take greater care than ever before in helping a student evaluate his own capacities: in knowing with some certainty that he will be engaged in that field in which he can make his greatest contribution to the war effort.

SHOULD SPECIAL GUIDANCE BE ATTEMPTED IN THIS WAR ERA?

With headlines blaring the need for women workers, and with women from all communities joining the parade to war industries, it might be assumed that girls are planning for their active part in this war effort.

If the girls in school have recognized this need for women in industrial and technical vocations, then there should be evidence of an increased interest in matters scientific and technical. Enrollment of girls in mathematics classes and science classes should approximate that of boys; they should be electing shop work and reading the popular technical periodicals. The differences between scholastic interests and preparation of boys and girls, so far as it has been due to differences in future plans, should tend to disappear. And yet, there is no evidence to show that such is the case. For all practical purposes, women are as unprepared to assume their technical role as they were when Hedrick[1] made his statement.[2]

It is crucial that these girls be aroused to the fact that they themselves will be taking an active part in this cataclysm so that they can save precious hours by securing adequate training before their call to duty.

To know that this war will continue and that the student of today will probably be on the battlefront tomorrow, to know that an older brother will not return immediately after armistice because of post-war conditions–these can easily give rise to serious emotional disturbances in the adolescent. Should the teacher risk such disturbances in order to lead these students to an adequate preparation for the active part they themselves will play? In answer to this question, Stanford's Dean Grayson N. Kefauver has said:

> The teacher has a responsibility for helping students realize their present responsibilities and their future responsibilities in the war effort. This policy is made desirable by the contribution youth can make in wartime and the contribution such teaching will make to the morale of youth and to the development of an appreciation of the inadequacy of war as a method for resolv-

[1] E. R. Hedrick, "Mathematics in the National Emergency," THE MATHEMATICS TEACHER, Vol. 35, No. 6, Oct. 1942, pp. 253–259. (Read Feb. 20, 1942 before the National Council of Teachers of Mathematics at San Francisco.)

[2] Acting Dean Lucien B. Kinney, School of Education, Stanford University; unpublished letter.

TABLE I

Vocations in Urgent Need of Additional Workers with a College Education

Health
Physicians*, △
Nurses*, △, +
Experts in public health*
Psychiatric-social workers*
Physiotherapists*, ‖
Occupational therapists*
Laboratory technicians*
Bacteriologists*
Chemists*
Pharmacists*, ‖
Dietitians*, ‖

Diplomatic Services and Special Investigators
=*Linguists**
Mathematicians*
=Secretaries*
=Journalists‖

Schools and Colleges
=Teachers*
=Nursery school experts*
=Psychologists*

Science
Physicists*, +
Chemists*, +
Chemical technicians‖
Geologists*
Metallurgists‖, +
Meteorologists‖
Mathematicians*
Agriculturists*
Home Economists*, △

Business and Industry
Engineers*
Electrical+, #
Chemical+
Mechanical+
Sanitary+
Architect+
Industrial Designers#
Telephone‖
Radio‖
Draftsmen‖, +, △, #
Mathematicians*
Engineering computations‖
Statisticians*
=Accountants*
=Secretaries*
=Personnel work‖
=Public administrator‖
=Economist‖

* Included in list of occupations in which workers are needed in *Higher Education and National Defense*, issued by the American Council of Education, Bulletin No. 35, Oct. 17, 1942.
△ Included on list of Mary Elizabeth Pidgeon, Chief, Research Division, Women's Bureau, U. S. Department of Labor.
\+ Civil Service examinations in these vocations open to women.
‖ Listed in *War Jobs for Women*, published by the Office of War Information, Magazine Section.
\# Listed by Arthur E. Wood, Area Supervisor, U. S. Employment Service, Los Angeles.
= Occupations requiring little background in science or technical mathematics.

ing international differences. The tragic waste of human and natural resources should compel a recognition of the great need for a collaboration of nations of the post-war period to insure a continuing peace. Education in wartime should be concerned with helping the pupils to gain an understanding of the issues in the war and post-war periods and a competence to deal adequately with them.[3]

Thus it appears that detailed guidance is needed and that it is the teacher's responsibility to give it.

WHAT VOCATIONS ARE IN GREAT NEED OF ADDITIONAL WORKERS?

In order that students may face their responsibility of preparing for a vocation useful in the war effort, we must know the occupations in which there is great need for additional workers. College placement executives state that three of the vocations in which additional workers are most urgently needed are engineering, nursing and medicine, and accounting.

The intensity of the need for engineers is shown by the following statements:

> If a school will send now one extra pupil to engineering training for every three it sent last year . . . all needs will be met.[4]

> A recent survey of the plans of 8 [airplane] companies operating in California shows a larger number of engineering college graduates desired before the end of 1943 than will likely graduate from the entire nation's colleges without Army and Navy commissions. And we are just talking about California companies.[5]

[3] Dean Grayson N. Kefauver, School of Education, Stanford University, unpublished letter.

[4] *Engineers Are Needed*, a pamphlet published by the Federal Security Agency, U. S. Office of Education, Vocational Division; p. 5.

[5] C. T. Reid, Douglas Aircraft Company, Inc., Santa Monica, Calif. "Engineering Job and Training Opportunities in the Aircraft Industry," an unpublished paper presented at the Texas Personnel Conference, University of Texas, Oct. 29, 1942.

Today our boys are being sent to the armed forces, not to colleges. It follows that our manpower reserve for engineering consists largely of women. This fact must be clearly recognized by the schools.

In nursing, the intensity of the need is shown by the fact that goals have been set for 1943–44 of 65,000 new nurses in training.[6] This means 2% of the 2,700,000 additional women needed as workers during 1943 will be nurses.

The following tables list occupations in which there is urgent need for additional workers. In vocations which require college training, it will be found that the majority involve a mathematical and scientific background. However, it is worthy of note that there are ten occupations in which college graduates with little mathematical background can fill an urgent need.

TABLE II

Vocations in Urgent Need of Additional Workers without a College Education

Semi-skilled occupations

Riveting#
Drill press operators# } in aircraft manufacturing
Final assemblers#

Clerical and stenographic occupations in Civil Service △

Listed by Arthur E. Wood, Area Supervisor, U. S. Employment Service, Los Angeles.

△ Included on list of Mary Elizabeth Pidgeon, Chief, Research Division Women's Bureau, U. S. Department of Labor.

TABLE III

Occupational Levels with Serious Labor Shortages[7]

Our shortage in	Percentage of executives who regard labor shortages as acute or serious. Is serious or acute	Will become serious	Total
Semi-skilled labor	56.7	12.1	68.8
Technicians	48.7	9.3	58.0
Unskilled workers	41.8	11.6	53.4
Supervisory personnel	35.2	10.4	45.6
Clerical help	28.4	10.1	38.5
Managerial personnel	18.5	6.4	24.9

[6] Subcommittee on Nursing of the Health and Medical Committee, Office of Defense Health and Welfare Services in *Professional Nurses Are Needed*, Vocational Division Leaflet No. 10, U. S. Office of Education; p. 2.

[7] "Fortune Management Poll," *Fortune*, Vol. XXVII, No. 2, Feb. 1943, p. 140.

Table III portrays the levels of greatest shortage of workers. A questionnaire was sent to 5,000 ranking business executives for this "Forum of Executive Opinion" as compiled by *Fortune*. It is noteworthy that over 50% of these executives anticipated shortages in semi-skilled labor, *technicians*, and unskilled workers. It is also interesting to compare the shortage in clerical help with that in semi-skilled labor and technicians.

How Great Is the Need for Women Workers?

Almost ¾ as many *additional* women will go to work in 1943 alone as entered industry during the entire 3–year period of January, 1940 to December, 1942. At the end of 1942 there were approximately 15,000,000 women at work. This was an increase of almost 4,000,000 over the 1940 total. In January 1943 the War Manpower Commission estimated that another 2,700,000 women would be needed in industry before December, 1943.[8] This is exclusive of the additional men needed.

In contrast to the number needed, only 2,500,000 non-working women would be willing to accept full time employment, according to a November sampling of the Bureau of Census.[9] Moreover, the availability of those willing to work is complicated by the fact that "approximately ⅓ are over 45 years of age and only ¼ would be willing to move from their present communities."[10] Furthermore, the War Manpower Commission has established a policy of importing as few workers as possible to areas with labor shortage. Instead, "resort to 'idle' women becomes the only way to compound fresh manpower without compounding services."[11]

[8] "Manpower Requirements in 1943" by the Bureau of Program Planning and Review of the War Manpower Commission; mimeographed report.

[9] "The Nation's Available Labor Reserve, November, 1942" by the Bureau of the Census of the U. S. Dept. of Commerce.

[10] *Ibid.*

[11] "The Margin Now Is Womanpower," *Fortune*, Vol. XXVII, No. 2. Feb. 1943 p. 222.

Thus, of the 12,600,000 non-working women available (i.e. able-bodied, and without children under 16 years of age) as reported by *Fortune*, only 2,500,000 would be willing to work. This might be compared with the fact that in England "all but 5,500,000 of the 17,250,000 women between the ages of 14 and 65 are in factories, uniformed, or civilian defense."[12]

A further fact in the need for women in this war effort is the change in attitude of industry toward the employment of women. Results of a survey of employer attitudes in July by the U.S. Employment Service show the percentage of all hiring in which women were accepted.[13]

	% of all hired who were women in Jan. 1942	% of all hired who were women in July, 1942
Professional and managerial	18%	63%
Skilled workers	16%	40%
Semi-skilled jobs	46%	70%

Also, the president of a large manufacturing plant stated:[14]

> Three months ago we had no women employees except the office staff; 25% of our total plant is now operated by women; it will be 75% within a year; it could be at least 95% if women were trained to take over jobs requiring scientific knowledge and technical skills.

Thus, there is a willingness to accept women, not only as semi-skilled craft workers, but even as scientists and technical workers.

New Vocational Opportunities

The list of vocations in tables I and II adds impetus to the fact that today industry is pleading with university placement bureaus to find women who can follow technical vocations. Moreover, for non-college women, the U.S. Employment Service is publicizing and dramatizing the need for women in crafts and the ease with which they can become semi-skilled. Recognition of women in industry is borne out by their acceptance into labor unions and by their promotion to supervisory positions.

a. *Engineering and Science in Industry*

It is probably most difficult for women to realize that they are really wanted as engineers and scientists. Regarding this as an occupation for women, there are two schools of thought represented by (1) those who know the present demand for women engineers; and (2) those who have watched women struggle again and again to find an opening in this occupation. First are the facts presented by those who feel engineering has become a new occupation in which women can pioneer with some hope of success.

(1) Such firms as General Electric and Westinghouse have urgently requested that college placement bureaus find women engineers to enter their "test course"—the internship of the engineer. These women would not work as sub-professional workers in drafting, etc. with little hope of advance to real engineering duties. Instead the companies are eager to find women who are able to undertake the more highly scientific phases of engineering work. Such women would be accepted without discrimination and with the same recognition and opportunity as that given men of equal training and ability.

(2) For the first time in history the U.S. Civil Service has opened its engineering examinations to women. More, certain examinations, such as that for Chemist, Junior Grade, are closed to men.[15]

(3) Curtiss-Wright is even paying for the training of 800 college women in 8 different engineering schools. Girls receive $11 per week with tuition, board, and room paid by Curtiss-Wright.[16]

[12] *Ibid.*

[13] "Womanpower: the Key to the Manpower Problem," *Employmeut Security Review*, Vol. 9, No. 12, Dec. 1942, p. 13.

[14] *Higher Education and National Defense*, Bulletin #35, Oct. 17, 1942, issued by the American Council on Education.

[15] War Jobs for Women, published by the Office of War Information, Magazine section, p. 10.

[16] "Higher Education and the War," *Journal of the American Association of Universily Women*, Vol. 36, No. 2, Winter 1943, pp. 93–94.

(4) Douglas Aircraft Company, Inc. of Santa Monica makes the statement:[17]

> The need for women trained in such subjects as mathematics, science, and precision skills cannot be stressed too highly. There are opportunities for women in almost every department of our company.

(5) U. S. Civil Service also is paying women college graduates while they train free of charge for a three-months period. At the end of that time they are given the rating of Junior Engineer with a salary of $2,000 per year.[18]

> (6) The Northwestern Conference on "College Women and the War" . . . recommended that industry and other interested groups develop realistic publicity on the need for educated, scientifically-trained women, and for short training in scientific fields as well as longer professional programs.[19]

(7) The increased willingness of industries to employ women in their professional and technical work is shown in the preceding section entitled "How Great Is the Need for Women Workers?"

On the other hand are those who have seen one woman after another attempt to enter the engineering vocation.

(1) According to Miss Mary Elizabeth Pidgeon,[20]

> There undoubtedly will be opportunities for women in engineering, but they will be relatively few. The woman going into this profession will have to have the initiative and ingenuity to make a place for herself in some type of work suited for women.

(2) Miss Olive Dennis, one of the few women engineers in 1940, stated:[21]

> If a woman tries to compete with men graduates in engineering on a comparable basis, I feel that she will be ignored. . . . Most large corporations like DuPont prefer men applicants because they feel they are more suited to advancement in any department of the company. . . . Unless a woman has a family connection in an engineering firm, or enough capital to go into business herself, her chances of rising to an executive position in structural engineering seem negative.

(3) Miss Mary V. Robinson states that today[22]

> Girls are mostly needed in scientific and technical fields only in an "assistant" category—as laboratory technicians, and technical aides.

For the present it appears that engineering is an occupation in which women are greatly needed, and in which they are being accepted, both on the professional and sub-professional levels. However, the future is problematical, but somewhat optimistic as noted below. Engineering for women is a pioneer field. It will probably be like medicine at the end of World War I. A very few women have already surveyed the route. (0.4% of all engineers in 1940 were women as compared with 4.6% of all physicians and surgeons who were women.) Many more will tread the route during the immediate future. It appears that a new profession is opening for women.

b. *Drafting and Mathematical Computations*

Miss Mary Elizabeth Pidgeon states that[23]

> Women's promise of going far in the future (in these occupations) is not great, but the practical contact with industry and the skill attained may be a stepping stone to further work . . . . The need for practical and applied mathematics is likely to continue after the war.

c. *Supervisors of Industrial Workers*

Miss Mary V. Robinson states that[24]

[17] Newton H. Anderson, Supervisor, Education Department, Douglas Aircraft Company, Inc., Santa Monica, Calif., unpublished letter.

[18] "Junior Engineer Course for Women," *Education for Victory*, Vol. 1, No. 21, Jan. 1, 1943, p. 12.

[19] "Higher Education and the War," *Journal of the American Association of University Women*, Vol. 36, No. 2, Winter, 1943, p. 92.

[20] Mary Elizabeth Pidgeon, Chief, Research Division, Women's Bureau, U. S. Dept. of Labor, unpublished letter.

[21] Olive Dennis, Engineer of Service, Baltimore and Ohio Railroad Co., unpublished letter to Women's Bureau, U. S. Dept of Labor on Feb. 29, 1940.

[22] Mary V. Robinson, Director, Division of Public Information, Women's Bureau, U. S. Dept. of Labor; unpublished letter.

[23] Mary Elizabeth Pidgeon, *op. cit.*

[24] Mary V. Robinson, *op. cit.*

Experience in unskilled jobs will always stand girls in good stead. Some of them will find eventual promotion to supervisory and even to personnel-administrative jobs.

Fortune states[25]

Girl supervisors on inspection lines are by now quite common, having had, as a rule, about a year's experience and training.

d. *Crafts—semi-skilled workers*

Acceptance of the woman worker by unions has been slow in coming, but officials of both the CIO and AFL have indicated their approval of admitting women to membership.

(1) For the first time in history a union opened its membership to women. In October, 1942 the International Brotherhood of Boilermakers admitted to membership 5,000 women who had been working in these trades.[26]

(2) In October the American Federation of Labor adopted a resolution on women:[27]

Every possible encouragement and assistance should be granted to unions seeking to enlist women in our movement.

(3) Philip Murray, Pres. of the Congress of Industrial Organizations states "The Congress of Industrial Organizations has no bars restricting the admission of women to its unions."[28]

What Will the Future Offer in These New, Specialized Vocations?

Whether or not a girl studies in the fields of science or engineering will depend upon two facts: (1) the intensity of her interest in the field; and (2) the future of women scientists and engineers after the war. It is not judicious to urge a girl to spend four of the most precious years of her life in becoming a scientist only to find that there is no place for her in the post-war era. There must be some probability of continued work in her career if she so desires.

There can be no absolute guarantee ever for the future years about any vocational or professional area. It is believed, however, that in the case of women any post-war excess of specialized training can be absorbed more easily than in the case of men, due to the fact that a goodly proportion of women will engage in professional activity during the war period to meet the emergency and at the end, in many cases, will desire to retire voluntarily from the specialized labor market.[29]

It is to be expected that prejudice against women in many professions will weaken as a result of the work women are doing in the war effort. The last war opened up new opportunities to women seeking technical or professional careers. Naturally, it would be impossible to give a quantitative measure of this decrease in prejudice or to assure women training for some type of work that they will be freely accepted in that line of work after the war. Such decisions depend on the post-war attitude of employers or employing agencies, of fellow employees, and to some extent on public opinion.[30]

All the experience now being gained in air cargo operation isn't going to be tossed aside on armistice day. Another thing we can see is the steady stream of travelers on official . . . business who commute by air between the capitals of the continents. . . . We don't have to wait for perfection of a poor man's airplane to find a non-military market for aircraft. There is an enormous market now for common carriers. Surely the engineers engaged in the manufacture of these have better than average chances of permanency and steady rise.[31]

It appears that a girl can hope for more opportunity in science and engineering than in the past; but she should undertake a vocation in these fields only if she has the personality and vitality as well as the talent to enable her to pioneer in a new field.

[25] "The Margin Now Is Womanpower," *Fortune, op. cit.*

[26] "Labor Organizations and Congresses," *Monthly Labor Review*, U. S. Dept. of Labor, Bureau of Labor Statistics, Nov. 1942, p. 1006.

[27] "Union Attitudes toward Women Workers," *Employment Security Review*, Vol. 9, No. 12, Dec. 1942, p. 14.

[28] "Union Attitudes toward Women Workers," *op. cit.*

[29] *Higher Education and National Defense* issued by the American Council on Education, Bulletin No. 25, April 17, 1942.

[30] Frederick F. Stephan, Chief, Estimates and Statistical Coordination Division, War Manpower Commission, unpublished letter.

[31] C. T. Reid, Douglas Aircraft Company, Inc., *op. cit.*

What Training Is Necessary?

Adequately to replace a man in any vital role, a woman must meet the same requirements as the man. To meet the slogan, A WOMAN TRAINED TO DO THE WORK OF EVERY MAN NEEDED AT THE BATTLEFRONT, it will be necessary for both the teacher and the girl to know definitely about all requirements for entering these new vocations.

The slogan means that the training that has been given previously to our boys is now needed by our girls:

Mathematics: The craftsman can USE as much mathematics as he can MASTER. This includes mathematics through calculus.
The scientist must have mathematics through trigonometry in high school.
The clerical worker needs MASTERY of arithmetic.

Science: the craftsman needs a general knowledge of the elements of the scientific world, and a detailed knowledge of the science which applies to the particular craft of the individual.

Industrial arts: the craftsman needs sufficient training in at least one field to become a skilled workman—a specialist.

In the past men have repaired automobiles. According to the slogan, women will have to be trained to do this. Men have been trained as carpenters who could construct gliders. Women will have to be trained as skilled carpenters. Women are becoming trained in the metal-working trades more than in any other; but they will have to become skilled metal workers: machinists, toolmakers, etc. This requires technical training and skill. Thus, besides continuing the work they have always done in the home and in the commercial world, women will have to assume these added competencies in the industrial world.

They will find work in three places: in the armed forces, in defense industries, and on the home front. In the women's reserves of the armed forces all types of workers are used: clerical workers, mechanics, scientists, etc. However, the greatest need is for specialists in some type of work, especially in technical and scientific fields. The SPARS (Women's Reserve of the U.S. Coast Guard) and WAAC have no special requisites. However, the WAVES demands that every officer have algebra and geometry, and the WAFS (Women's Auxiliary Ferrying Squadron whose members are non-military Civil Service employees of the War Department) requires algebra, geometry, trigonometry, physics, two years of college training, and a pilot's license. Thus, preparation for entering the armed forces will usually mean (1) either studying as much science and mathematics as the girl is able to master; or (2) becoming highly skilled in some occupation.

In defense industries preparation includes all that has been undertaken previously by men: from elementary skills in wood or metal shop and arithmetic, to the precision skills and mathematical knowledge of the skilled craftsman such as the machinist, and to the scientific and mathematical knowledge of the engineer or scientist.

On the home front work is usually of the commercial, social, or home arts type which women have been handling along with men for the past two decades. Preparation for this field will continue much as it has in the past, with the exception that workers will be accepted on jobs with far less training than previously.

Should a Girl Change Her Vocational Plans?

Adequate counselling must take into account the needs of the individual as well as the needs of society. This raises many disturbing problems for the counsellor.

As the girl of today first realizes that hers is the responsibility for meeting manpower needs of industry, as well as commerce and the home front, she thinks of those fields which have been most widely

publicized and dramatized: the crafts and clerical work. She wonders if she should change her career plans so as to become skilled in one of these fields: specifically, if the girl's chosen vocation is that of teacher of social studies, or journalist, or commercial artist, should she change her plans to include some industrial craft or some technical knowledge?

To advise on such questions, it would be helpful to the teacher to know (1) the percentage of all women workers who will be needed in the main vocational categories; and (2) what part of each vocational group could be secured in the various age levels of all women without further training. However, the War Manpower Commission and the Department of Labor both report that such information is either unobtainable or confidential at this time.

Although there are vocational fields in which greater need is reported than in others, both the Department of Labor and the War Manpower Commission concur with Miss Thelma McKelvey in her statement:[32]

> In the list of essential activities you will find very few fields omitted. I do not think that we have yet accomplished equal opportunities for women. Our educational methods should include more along this line . . . . In view of the war needs, women who have training, education, and who are willing to work will find unlimited opportunities in practically every field.

However, with manpower so crucially short, it is vital that each individual be working up to his highest capacity. This can be done only if the girl is guided into the vocation in which she is interested, happy, and successful. Here, then, is one of the greatest responsibilities of the teacher today: to guide these girls into occupations in which they will make a successful contribution to the war effort.

If the girl has the talent, interests, and background necessary to be SUCCESSFUL as an artist, a teacher of social studies, or a journalist, then she should be encouraged to follow her vocational choice. If not, it becomes the responsibility of the teacher to help her evaluate herself and choose a war-adapted occupation in which she will succeed. Whatever her final choice, the girl should be led to a knowledge of the ways in which that type of work can be of use in the war effort. This is the task of each individual teacher as he presents his own subject.

Should a Superior Girl Take Time to Study Science or Engineering If She Is Interested in These Fields?

Teachers find that when the responsibility for work in the war effort is presented to superior girls, their first reaction is to drop their studies and enter a defense industry as an unskilled, or semi-skilled worker. The solution to this problem is not easy; it must be given individually with the needs of each particular girl in mind. Officials in the War Manpower Commission, the Women's Bureau of the U. S. Department of Labor, and the U. S. Employment Service, however, all concur that superior students interested and talented in scientific and technical occupations should be challenged to assume as their part in the war effort the period of scientific and mathematical training required for technical work.

Colleges agree with this point of view:[33]

> A national shortage of trained minds is a serious threat. In the secretarial field high school students seem well apprised of shortages; but in many others requiring the liberal arts education they are not so well informed. It is clearly the duty therefore of schools to do more effective educational and vocational guidance.

To meet these conditions, first of all the superior girl needs to be challenged to her place in the war effort: not the semi-skilled work that any can do; but the scientific

[32] Thelma McKelvey, Principal Assistant, Women's Program of the War Manpower Commission, unpublished letter.

[33] *Higher Education and National Defense*, issued by the American Council on Education, Bulletin No. 25, April 17, 1942.

or engineering work that only the gifted student can understand—work in which shortages are just as acute as in the craft work. In working with girls who are talented in mathematical and scientific studies, the teacher's task becomes not only arousing girls to definite preparation for work in this war effort; but more, it becomes one of building a desire to assume those highest responsibilities for which the student's talents and interests fit her.

Summary

Whether womanpower is drafted or voluntarily enlisted, it is certain that single women—the girls in school today—will be working in defense industries or essential work as soon as their school training is completed. This in turn means that it is the responsibility of the teacher to make sure that each girl is skilled in some work that is needed in this war period.

Within this problem of vocational guidance for girls lies one of the greatest responsibilities of every teacher in this war effort. Above all else stands the challenge to guide these girls:

(a) To a recognition of their coming active part in the war effort;
(b) to a planned preparation for skill in some specific war work;
(c) to a knowledge, both of the multitude of vocational opportunities, and of the dire need for workers with scientific or technical training in such vocations as engineering, chemistry, nursing, and medicine; and
(d) to an analysis of their own talents and interests in relation to the vocations in which workers are needed.

Most crucial of all is the challenge to teachers to help girls select a vocation and plan their program of studies so that their abilities will be used to the maximum. With three of the greatest manpower shortages in engineering and science, nursing, and medicine, it is important that girls with ability and interests in these fields be guided there for study rather than be allowed to wander into craft work through ignorance of the opportunities in the other fields.

Today, as always, is placed in the teacher's hands the responsibility for planning with the student so that she will achieve a balanced womanhood: skilled in the duties and obligations of her home, and skilled in some vocation as a life career. But now an even greater responsibility is placed in the teacher's hands: to plan both for the individual's welfare and for the need for workers in this manpower crisis. It is the obligation of the teacher to see that each girl has chosen a life career so that she can be of maximum value in this war effort.

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What the Word "Function" Means to Algebra Teachers*

By LEE J. CRONBACH
State College of Washington, Pullman, Washington

TEACHERS are well aware of the fact that the pupil who can repeat the definition of a word may not really understand what that word means. Since understanding of technical terms in a subject like mathematics is essential, it is important for the teacher to determine whether the pupil has really mastered basic words. In one of a series of studies, the writer sought to construct a test which would determine how well pupils understand the word *function*, a term generally considered basic in work in advanced high school algebra and college mathematics. While attempting to build the test, it was found that teachers often did not agree as to whether a given expression should be called a *function;* this suggested that it might be important to determine just what is being taught as the meaning of *function*, since agreement as to what is being tested is necessary before a test can be constructed. The present article reports an attempt to determine what typical teachers of algebra mean when they speak of a function.

MEANINGS OF *Function* ACCEPTED BY MATHEMATICIANS

It may immediately be argued that no difference of opinion as to the meaning of the word *function* can arise, since the word has been carefully defined. A careful summary of many classic definitions has been presented by Hamley;[1] when these are compared it is found that even they disagree as to what may legitimately be called a *function*. Thus Bernouilli, Leibnitz, and Euler required that a function be an analytic expression or equation, while Dirichlet, in a later period, expressly stated that in defining *function* "we need never think of a dependence expressed in terms of mathematical operations." Hamley himself defines *function* as a "correspondence between two ordered variable classes,"[2] thus apparently ruling out relations between qualitative, unordered, variables. Some definitions restrict the word *function* to single-valued functions, while others do not, and so on.

Even apart from what mathematicians accept as an ideal definition of *function*, there is another significant question: what meaning is, and should be, used in the high school classroom? Thus Lennes proposes that the word be defined for mathematical purposes as follows: "Any mathematical expression, containing a variable x, that has a definite value when a number is substituted for x is a function of x." He adds, however, that "for the purpose of secondary mathematics we may say that a function is a quantity which varies in a definite way as some quantity involved in it varies."[3] He points out that thus broadening the definition permits one to refer to the amount of crime as a function of unemployment, and to accept other statistical relations as functions. Leitzmann, also, indicates the advisability of extending the definition of *function* to include statistical correspondence.[4]

* The writer is indebted to the many teachers who have assisted this study by their responses and suggestions, and especially to Dr Maurice Hartung for aid in planning the study.

[1] Hamley, H. R. *Relational and Functional Thinking in Mathematics*, pp. 13–21. Ninth Yearbook, National Council of Teachers of Mathematics, New York, Teachers College, 1934.

[2] *Ibid.*, pp. 4–6, 20.

[3] Lennes, N. J. "The Function Concept in Elementary Algebra," *The Teaching of Algebra*, p. 55. Seventh Yearbook, National Council of Teachers of Mathematics. New York: Teachers College, 1932.

[4] Leitzmann, W. "The Function Concept and Graphical Methods in Statistics and Economics," *ibid.*, pp. 74–92.

Construction of Test

To measure one's knowledge of a word the tester cannot merely ask him to state the definition he accepts; many of our concepts are not formulated in our minds as definitions, and even persons who have learned a definition may not apply that definition properly in attacking new problems. One characteristic of the learner who has mastered a word is that he can use it to name situations to which it applies but does not apply it to situations where it is inappropriate. If one were to present such correct and "false" illustrations of a word to a student, his ability to distinguish between them would to some extent indicate how well he knew the word. The construction of tests of this kind has been described elsewhere.[5]

A large number of true-false statements were prepared representing illustrations of the term *function*, and illustrations which might be mistakenly named "functions" by students. These statements were based on textbook definitions and illustrations, and on errors made by students. In order to determine the clarity of these items, intended for use in testing students, groups of teachers attending summer sessions and workshops were asked to mark them as they thought correct. As teachers disagreed in marking the items, they were discussed orally to determine where the disagreement arose; the debates showed unmistakably that these teachers are not using the same meaning for the word *function*. To obtain more systematic evidence of this conflict, the test was revised for use with teachers. Items were added to represent differences in definition appearing in the discussion of the first test. The items were answered by further teachers to eliminate poorly phrased or ambiguous items. Some difficult, but unambiguous items, were retained.

[5] Cronbach, Lee J. "An Analysis of Techniques for Diagnostic Vocabulary Testing," *Journal of Educational Research*, Vol. 36, November, 1942, pp. 206–217.

Cronbach, Lee J. "Measuring Knowledge of Precise Word Meaning," *ibid.*, in press.

Administration of Test

A final test of fifty-five statements was prepared and mailed to one hundred and forty-seven mathematics teachers. Thi group was selected by taking three names at random from the list of members of the National Council of Teachers of Mathematics from each state and the District of Columbia. Replies were received from forty-one persons, of whom twenty-eight had taught advanced algebra recently. Results reported below are based on the latter group. While this sample is small, it is considered significant, since results from the preliminary discussions led to similar conclusions. These teachers are probably superior to the average, as they are members of a professional organization and are sufficiently interested to return the questionnaire. On the questionnaire, each person indicated his school, whether he had taught advanced algebra recently, and the text used. In a covering letter, these directions were given: "Mark the test as you think correct, using the meaning of the word 'function' which you would desire your students to have at the end of a second year in high school algebra."

Extent of Disagreement among Teachers' Concepts

While there is general agreement among the teachers replying, on many seemingly basic points teachers disagree. On twelve of the fifty-five items, the division of the group between the answers "true" and "false" was nineteen-to-nine or closer; that is, more than one-third of the teachers disagreed with the majority. Some of the items on which disagreement was pronounced are the following (numbers in parentheses indicate number of teachers marking "true" and "false," respectively):

17. If $y=2x$ when x is less than 5, and $y=4x$ when x is 5 or greater, then y is a function of x. (14, 11)

34. The upkeep cost of an automobile increases as it grows older, but we do not know how to predict this by a mathematical equation; the upkeep cost is a function of the car's age. (13, 14)

39. If it takes 2 ounces more sugar per gallon to make blackberry preserves than peach preserves, the amount of sugar needed is a function of which of the kinds of fruit is used. (15, 13)
54. If the graph at right shows the amount of fruit borne by a certain tree at each age, the amount of fruit is a function of the tree's age. (Graph not reproduced here shows an irregularly rising broken line.) (11, 15)

It will be noted that many of the items in the test deal with "life situations" rather than purely mathematical expressions; this is a reflection of the point of view that the best instruction teaches concepts so that the pupil can apply them outside the classroom.

A simple measure of the amount of disagreement among teachers was obtained by making a key based on the response for each item accepted by the majority of the group. (This does not mean that the *correct* meaning for a word is best determined by a majority vote.) When the twenty-eight papers were scored by this key (omissions not being counted), the number of disagreements between an individual paper and the key was as high as fifteen out of fifty-five. The median number of "errors" was eight.

A further analysis was made to determine if these disagreements resulted from the use of different texts. Seven of the teachers used the Wells-Hart text; when a key based on the majority of this small group was prepared, the number of "errors" on the seven papers ranged from ten to three, median seven. A similar study based on seven cases using the Edgerton-Carpenter text showed a range of scores from seven to twelve, median eleven errors. Even more striking evidence that teachers using the same text may be working with concepts different in part is provided by answers of two teachers from the same school who happened to be included in the sample. They disagreed on four items, and on seventeen other items one or the other failed to respond. One of the items on which they disagreed is:

2. If $a = bc - 4$, a is a function of bc. (26, 2)

One further comparison may be made to demonstrate the extent to which one teacher may be presenting ideas which another teacher would mark incorrect: two teachers in different schools using the Stone-Mallory text disagreed on seventeen items. It may be mentioned in passing that similar disagreements among men teaching sequence courses in the same college mathematics department had gone unrecognized until brought to light during a discussion of this test.

An important question is whether the disagreements found are trivial or not. In general, it appears that disagreements are fewer on the more basic illustrations. (Thus, a graph of the familiar weight-postage problem was unanimously accepted as illustrating a function.) On the other hand, for even two teachers to disagree with the group on Item 2 (above) implies that their students may be handicapped. A few of the other items dealing with vital aspects of the concept which brought some disagreement are:

3. $36w$-1 is a function of w. (20, 8)
8. If s is greater than $4t$, s is a function of t. (4, 24)
35. If the amount of whole wheat in bread (w) and its vitamin rating (v) are related so that $v = 0.16w + 1.5$, w is a function of v. (20, 8)
43. If this table shows the amount of gasoline used per mile at different speeds,

Speed (miles per hour)	5	10	15	20	30	40	50	60	70	80
Gasoline per mile (gal.)	0.10	0.08	0.07	0.05	0.04	0.05	0.06	0.08	0.12	0.14

The gasoline used is a function of the speed. (15, 11)

Consistency of Teachers' Concepts

Waiving the question whether teachers use a correct meaning for *function*, one may ask whether, whatever definition they follow, they use it consistently, seeing its implications. An "inconsistency" key

was prepared by noting all pairs of items such that if a testee marked one of the pair true, he must mark the other true also to be logically consistent, or vice versa. These pairs included all items which were interchangeable, that is, which represented the identical type of function. One such pair is Number 8 (above) with

16. If y is less than $2x$, and greater than $x+1$, y is a function of x. (7, 19)

Although mathematically these seem similarly indeterminate relationships, one person marked Item 8 true and Item 16 false, while four persons rejected Item 8 as a function but accepted Item 16. Another such pair is

13. If $d^2=f+1$, d is a function of f. (24, 4)
51. If the graph at right shows the relation between a and b, then b is a function of a. [The graph, not reproduced here, was of the general form: $(b-k)^2=c-a$.] (24, 4)

Twenty-one teachers accepted both of these, one rejected both, but three accepted the first and rejected the second while three others rejected the first and accepted the second. Such differences are difficult to explain except as inconsistencies.

Each paper was scored by the inconsistency key, which noted thirty-three possible inconsistencies, of which, however, some were obvious contradictions. The maximum score was seven inconsistencies, with a median of two. These results suggest that teachers in this group have fairly precise concepts of the word *function*, so that differences between teachers' responses are in general due to true differences in meanings rather than carelessness and similar factors. It is nevertheless not entirely satisfactory that teachers of mathematics, a presumably exact science, should show any inconsistency in their responses.

Construction of Criterion Keys

Although each teacher had a reasonably precise concept associated with the word *function*, different teachers had different concepts. While a careful analysis of each individual's responses, item by item, yields a picture of his concept, a more efficient procedure is needed for studying many papers. The disagreements between teachers are centered on several major aspects of the concept, and by determining where each individual stands with relation to each of these aspects, a close approximation to his concept can be obtained. The preliminary discussions with teachers revealed a list of major points on which differences arise; each of these points is in essence a criterion by which the teacher decides whether an expression is a function. One such criterion, for example, is the test: For x to be a function of y, the relation must be expressed explicitly, thus: $x=f(y)$; in that expression, y is not a function of x. In the test are several items which would be rejected by this criterion; a key was made by noting all items which apparently would only be rejected on this basis. To increase the validity of this inference, pairs of "crucial" items were used where possible. These were pairs of expressions that were mathematically equivalent except in this one respect, that one satisfied the criterion in question and one did not. For example, these items constitute a crucial pair for the criterion stated above:

4. If $x=y$, then x is a function of y. (28, 0)
12. If $m-n=0$, then m is a function of n. (26, 2)

If a person marks Item 4 true and Item 12 false, it is probable that he includes the criterion of explicitness in his concept of *function*. While one such pair of responses might arise from some basis other than acceptance of the criterion, it is believed that the inclusion of several items or pairs in each key made interpretation more valid.

Elements in Teachers' Concepts of Function

The criteria for which keys were made, together with the responses of the teachers, were as follows:

A. It is necessary for the variables to be related by a determinate correspondence. Twenty of the teachers accept this criterion, making scores of five or more out of seven points, the maximum on this key. Two reject the criterion, scoring one. Six, with intermediate scores, appear uncertain. (Item 16, above, was typical of this key; marking the item true suggests rejection of the criterion.)

B. The relation between the variables must be systematic, such that its graph would be fairly smooth. Seven teachers accept this criterion, while eleven reject it. The distribution is bimodal, with eleven intermediate cases. (Item 54, above, was typical of this key.)

C. The relation must be single-valued; for each value of x there must be only one possible value of y. Apparently no teacher accepted this criterion. The key consisted only of two crucial pairs. These pairs included items 13 and 51; it was mentioned above that some teachers marked these inconsistently.

D. Not only must the variables be related systematically (Criterion B), but this relation must be expressed by the same mathematical expression throughout the domain of the function. Two persons accept this criterion, twelve reject it, while the remainder reject only one of the two items in the key. (One of them is Item 17, above.)

E. The variables may be no more related than by a statistical correlation. (This is contradictory to criterion A; however, one may accept Criterion E without considering such an indeterminate relation as shown in Item 8 a function.) Of three items of this type—one deals with the relation between intelligence and grades—five teachers accept two or three as functions, while eighteen reject all of them.

F. There must be a logical causal relation between the variables (if they represent real entities). Seventeen of the teachers accept this as a criterion, four reject it, and several omit items in this key. One item from the key is

> 31. If a certain man's weight starts at 150 lb. and increases 6 pounds each year between the ages of thirty and forty, and his wealth starts at $1000 and increases $250 each year, during that period his wealth is a function of his weight. (2, 22) (It will be noted that this is a single-valued determinate correspondence, and satisfies most accepted definitions of *function*, yet is not accepted by these teachers.)

G. The related entities must be variables. This criterion appears to be used by everyone in the group to reject some items, but three teachers are consistently willing to mark items of this type true:

> 29. The number of inches in a foot is a function of the number of inches in a yard. (5, 21)

A determinate correspondence where all values of the dependent variable are the same is not accepted as a function. (0, 28)

H. The variables must be quantitative, may not be such things as sex, state, or kind of fruit (Item 39, above). Two persons reject this criterion, and the position of two persons is uncertain; the remainder appear to accept the criterion in some situations. Twelve persons mark false such items as:

> 47. The table shown in part at right [not reproduced here] tells the number of automobiles registered in each state least year; the number of automobiles registered is a function of the state. (2, 25)

but accept Item 39, above.

I. The relation must be stated explicitly, not implicitly. Six teachers accept this criterion, while twenty clearly reject it.

J. The variable may itself be a function, such as $x-2$, or bc in Item 2, above. One teacher rejects this criterion, while two indicate uncertainty by omissions. The remainder accept the criterion.

K. A quantity may be a function of two variables at once, as "the area of a rectangle is a function of its height and length." This is accepted by all but two teachers; these appear inconsistent, hence possibly uncertain, on this key.

L. If $z=f(x, y)$, $z=f(x)$. This is clearly accepted by two teachers and rejected by nineteen; the responses of the remainder leave their opinion on the point uncertain.

Some of these analyses have been based on small numbers of items, as it was considered undesirable to lengthen the test in this study sufficiently to make the categories longer. Nevertheless, since the categories are quite homogeneous, results are probably trustworthy. Reliability coefficients were estimated by the Kuder-Richardson method for a few categories; that for category A, with seven items, was 0.50, while that for category H, with only four items, was 0.65. For a test where more

precise data on individuals are desired, longer categories would be required.

Relation of Teachers' Definitions to Their Responses to Situations

Some may question why such an elaborate test is required when the teachers could have been asked outright, whether or not they accept each of the criteria. The answer is shown by an analysis based on the one definition-type item in the test:

18. If a problem contains two variables, and when we know one we can find the other, the second is a function of the first. (26, 2)

Of the twenty-six who accepted this statement, seven marked false one or more items out of the first seven illustrations that were clear algebraic examples of the function described in Item 18. In other words, even with teachers, the verbal statements accepted as definitions are often not descriptive of the way the individual applies the term in question.

Summary

It has been shown that, insofar as this sample is representative, teachers of advanced algebra disagree widely as to just what can be called a *function* and what cannot. These teachers disagree in some respects with definitions of *function* accepted in mathematical analysis. Nevertheless, each teacher appears to have a fairly precise concept of the word function, even though teachers within the same school may have different precise concepts. Some teachers disagree with the majority even on points which appear basic in a useful concept of *function*.

A composite picture of what the word *function* means to these teachers would include these points: The variables must be related by a determinate correspondence, such that for every x there is a fixed value of y; the correspondence may be multiple-valued, but may not be statistical. The relationship need not be regular, such as could be expressed as an equation of low degree, nor does it need to be expressed explicitly. If the variables are taken from life situations, changes in the dependent variable must be caused by changes in the independent variable; mere co-variation is insufficient. The quantities involved must be variables which are not held constant during the problem. A quantity may be a function of two other variables, but if $z=f(x, y)$, one cannot say that "z is a function of x." Individual teachers may accept all, none, or merely portions of this composite "definition" for *function*.

No study of this type can pretend to determine what concept of *function* should be the goal in secondary education, but it may be questioned whether the concept used by these teachers is the most desirable. Lennes and Leitzmann, it was noted, suggested that the concept should be broad, including even statistical correspondences, but the present data show that teachers have not adopted this view. Probably any mathematician would insist that a functional relationship exists where a determinate correspondence exists, even if there is no direct cause-and-effect relationship between the independent and dependent variables; the confusion of causal relation and correlation is a serious error made by most consumers of everyday statistics. Nevertheless, a large majority of the teachers in this sample apparently hold that a functional relation must be a causal relation. Some of the other responses, such as the rejection by one-fourth of the teachers of the statement "$36w-1$ is a function of w," seem to indicate a concept that has limited value for studying variation.

The implications of these findings for education are clear. The pupil will of course be handicapped in applying his concept of *function* if his concept is not precise.[6] His concept cannot be precise if his

[6] This is reflected in the errors found by Boyd in her analysis of students' errors. (Boyd, E. N. *A Diagnostic Study of Students' Difficulties in General Mathematics in First Year College Work.* Teachers College Contributions to Education, No. 798. N. Y.: Teachers College, 1940. Pp. 152. See esp. p. 22.) She notes the interrela-

teacher and textbook have different concepts, or if his various teachers use different meanings for the word. Where students pass from one teacher to another in sequence courses, this may be a source of serious confusion. It may be suggested that teachers working together discuss the term *function* to determine just what concept each now has; a test of the present type may be a useful tool for detecting differences. Agreement on a workable uniform concept may be the next step.[7] Textbook writers should become aware of this as a source of possible difficulty and take great pains to see that their definition does not become a mere sound-pattern of little-understood words. The responsibility of many curriculum makers, who have, in the past, merely decided to "teach the function concept," is great. It would appear that new materials must be carefully interpreted to the teacher, lest each one, trying sincerely to teach the concept desired, be teaching her own misconception.

It must not be assumed that these difficulties are peculiar to mathematics. Mathematics may have other terms where teachers' concepts are as divergent as with the word *function*. But it is far more likely that subjects less precise than mathematics, subjects whose vocabulary has developed in recent years, present greater vocabulary stumbling blocks than does algebra. Certainly, it appears that the meanings held by teachers may be an important factor in vocabulary problems. Research to identify those words where teachers disagree, and, if possible, to secure agreement on a most appropriate meaning, is greatly to be desired.

tion of such difficulties: "Many students are not able to distinguish constants from variables and consequently they do not understand the function concept." (*Ibid.*, p. 116).

[7] This discussion does not mean to imply that teaching the function concept should be limited to clarifying vocabulary difficulties. Knowing the word *function* is but one aspect of an understanding and appreciation of co-variation.

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Can Arithmetic Be Correlated with the Unit of Study?

By HATTIE MAGUIRE
Montecito Union School, Santa Barbara, California

FIRST TEACHER: "Well, you seem to be able to correlate arithmetic with the unit, but I couldn't possibly get enough arithmetic out of my unit to satisfy my parents."

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THIRD TEACHER: "The book puts it so much easier than I could ever make it, so why make arithmetic harder than it is for the poor kiddies?"

So speak teachers on every side who have not understood what correlating arithmetic with the unit means, and have not actually tried for a period of time to correlate it. The purpose of this paper is to answer these questions from the standpoint of one who taught from the "good old textbook" and who finally conscientiously tried to correlate all the arithmetic with the unit, and found to the surprise of all concerned that it actually worked. It is easy and meaningful to the children. The textbook is not discarded, but is used as another reference book just as the encyclopedia and dictionary are used.

In order to convince you, too, that arithmetic can be correlated with the unit, I shall use the following outline:

1. Opportunities for all skills to function provided by the activity program
2. The practical value of arithmetic
3. Adequate justification for arithmetic in the curriculum
4. Characteristics of improved method
   a. Time allotment
   b. Textbooks
   c. Teacher guidance
   d. Pupil needs
   e. Problem solving
   f. Individual help
   g. Practice
5. Examples of practical application
   a. Problems
   b. Method of solution
   c. How to write problems

1. Opportunities for Skills to Function Provided by the Activity Program

"To capitalize on the best in the new program and yet to conserve what the test of time has proved worthy in the old is the main job of the modern school.

"The modern school sets up conditions that will provide the best possible pupil growth in various skills and techniques. Skills are not taught as ends in themselves, as separate and direct objectives, but instruction in skills is given to provide children with techniques for genuine functional purposes. The modern school is not interested in skills, *per se*, as in skills that function when children have some purpose for using them.

"In the modern school skills grow out of activities and take their meaning from experiences. (Example: The class decides to build a lobster trap, so the children must learn to measure and follow directions.) The meaning and purpose come first. Skills, therefore, develop from an activity program and are an integral part of it. Life necessities and comforts, rather than abstract subject matter, are found to be sufficiently significant to be the basis of the new curriculum. An activity program provides a functional background for the 'three R's.' Nothing is included that does not have functional value in the child's life. *Genuine learning demands* rather than formal processes are recognized as the more appropriate curriculum foundation. The best progressive practice achieves a balance between formal instruction in skills and a rational program of educational worth-while activities. Therefore, everything is done by the progressive teacher to enrich the program,

and to give meaning to the activities, and to provide curiosity and interest in learning and findings facts. The progressive teacher utilizes all the features of the child's immediate interest and environment that contribute to the use of skills. These provide experiences, activities, discussions. Now effort is unconscious, attention is a normal function of the situation, and discipline problems disappear when learning is related to living."[1]

2. The Practical Value of Arithmetic

Now skills are introduced more gradually as well as more informally. Modern and progressive teachers first determine aptitudes and preparation before instructing their pupils. Instead of making arbitrary learning demands they guide their children's development. Each teacher shifts from the formal instruction of a passive pupil to a guidance expert, an investigator, and director of the learning process. Modern teachers disregard any preconceived notion of what "grade four" means, and instead teach in terms of the needs of children.

Educators have now revised their concept of what constitutes desirable outcomes in skills. The question as asked formerly was: "How far has this child gone in arithmetic?" Now, we ask, "How well can he use his newly acquired skills?" and, "Has he acquired automatic and efficient habits? Is his arithmetic computation effective enough to be of any practical use? Can he solve a real problem?"

Evidence has been presented by scientific studies of arithmetic curriculums that much of the traditional subject matter commonly taught in the schools of the past generation is of little social utility, and much too difficult for many of the children in the grades in which it has been taught. So useless processes and topics have been eliminated, thus reducing the complexity of the skills that the pupils are now expected to acquire.

[1] Hildreth, Gertrude, *Learning the Three R's.* Philadelphia: Educational Publishers, Inc. 1936. pp. 1–5.

3. Adequate Justification of Arithmetic in the Curriculum

We are coming to treat arithmetic in a more meaningful way, where the child acquires skills and develops reasoning abilities naturally. Arithmetic is justified in the curriculum only insofar as it prepares the child to meet his immediate and future arithmetic needs, and all those skills are in some form of *problem* situation. The mastery of skills is required only because it contributes toward this major aim.

4. Characteristics of Improved Method

a. Time Allotment. Time limits for skills have been greatly reduced. Modern school systems use the time formerly allotted to formal drill work in a somewhat different way: it may shift about during the week; it may be lengthened, shortened, or be redistributed, depending upon the rest of the program and its relative importance.

b. Textbooks. The textbook has a somewhat different place in modern schools. We seldom speak of a basic arithmetic, but instead we provide suitable material in many different forms appropriate for the child's learning level such as; charts, children's own drawings, scrapbook collections, diagrams, graphs, construction work, loose-leaf materials, work sheets, and games.

c. Teacher Guidance. Teacher guidance —not teacher control—is the modern school's slogan. She uses the child's cooperation and self-motivated reaction. Less of the teacher's time is spent in thinking for the children, and more of the time is spent in arranging the situation so that the children act for themselves under their own impetus. She places the responsibility for learning on the child. Then she stimulates him to make more effective use of his time.

"The newer concept of the teacher is that of a guide in child learning, a sharer of the child's learning experiences, a diagnostician when things go wrong, and a remedial expert to correct faulty learning. This is asking a good deal of modern

teachers, for the newer approach is less simple than demanding with a ruler in hand that the children 'now turn to page twenty-nine and do each example in succession'."[2]

d. Pupil Needs. "It is also fortunate that there is seldom a conflict between individual and social need. What is good for the individual is frequently good for the social whole. The individual himself is very much in need of gaining what society has to impart through the written word. So reading, writing, spelling, and arithmetic have a common meeting ground in the needs of the child and the needs of the community."[3]

"It is the teacher's work to watch for the developing needs of the pupils. As the student comes to the teacher, it is necessary for her to estimate and analyze his needs. Nor is it to be interpreted in any narrow sense. Teachers have sometimes been misled in their understanding of the meaning of the word 'need' as an organic beginning point for a unit of conduct. In the first place, needs have sometimes been confused with merely felt needs. Thus teachers have waited until the children have actually expressed in words some specific need before they have been ready to do something about it. This last procedure may result in the unwarranted delaying of the teaching of many things which the children need. Teachers should reveal needs to the children. By a few suitable and well chosen questions or suggestions teachers may reveal to the children their own need and suggest ways and means of satisfying it. It is the teacher's privilege to reveal to the children their needs. Furthermore, it is essential that teachers assist children in evaluating their felt needs and that children may be helped to desire their best needs."[4]

e. Problem Solving. With the more informal program in the early grades, children are not rushed into such early contact with complicated number facts, and the pressure for accomplishment during the formative years is greatly relaxed. It is surprising to see how many natural or life problems arising from daily experiences can be solved satisfactorily by the child who has had only informal practice in number computation. Any learning gained by a child in the quest for the solution of a natural problem is much more vivid and more important to him and, hence, more permanent than when gained in the traditional arithmetic class. In participating fully in the experiences of the classroom the pupil learns the meaning of numbers, the visual and auditory symbols for numbers, and the everyday usage of these concepts. Firsthand, genuine problems requiring the use of arithmetic are continually arising in all classrooms. They appear without regard to any previously arranged course of study. For example, the class may meet a problem involving multiplication, and the course of study or textbook may suggest that multiplication belongs in another grade. What should the teacher do? She should encourage the situation to help the pupils understand the *meanings* of multiplication. The teacher here distinguishes between the teaching for meaning and the teaching for skill in a process. Every situation providing an opportunity for the building of meanings should be utilized whenever it arises, regardless of the course of study, because meanings and concepts are broadened and fixed through varied experiences. Probably the process will not be mastered at this time, but an *acquaintance* with it will be made at this time, and the knowledge taught later.

The solution of many problems proceeds not by trial and error and a fumbling about with obscure material or data until by chance the right process is hit upon, *but is achieved through thinking, reflection, the development of insight, and through a consideration of the features of the problem and their relationships.* The learner sees after a thoughtful pause the implications of the conditions existing in the learning situation, or through previous experience, and

[2] *Ibid.*, p. 14.
[3] A. Gordon Melvin. *The Activity Program.* New York: Reynal and Hitchcock, Inc., 1936, pp. 28 and 29.
[4] *Ibid.*, p. 26.

relates it to a problem previously solved.

Learning through seeing meaningful relationships is always more possible when:

1. The learner is more accustomed to meeting problems through thoughtful reflection; when he reflects or contemplates the problem before undertaking to solve it.
2. When he is fairly well acquainted with the problem or finds in it a familiar setting, when many of its features are already familiar, previously practiced, or when relationship is seen between this new problem and previously practiced ones.
3. When both the problem and its goal are within the perceptual or comprehension range of the learner, or through instruction can be brought within his range.

How can the teacher prepare her children so they will be able to surmount difficulties and avoid frequent failures?

1. "Recognize problem situations. Choose problems which arise in the classroom. They will be vivid. Encourage the children to form the habit of imagining themselves in the situation.
2. "*Reading for comprehension.* Teaching children to read with comprehension is a major aim of all teachers. A good method is to have the class read silently the problem. Then practice telling in his own words the story of the problem. As a means of retaining comprehension of problems many teachers have their children solve problems by the 'formula' method. That is, they set up a series of questions which the children answer before they attempt to solve the problem.
3. "*Correct habits in comprehension.* The teacher can introduce and teach processes in meaningful situations; she can rationalize processes where it seems advisable; she can adapt all practice to individual needs; she can diagnose each child's difficulty; she can plan remedial practice for those difficulties.
4. "*Choosing the correct operations.* The meanings of the various operations must be taught continuously; the teacher must use a varied vocabulary in asking questions which require the use of any one process in finding the answer, simplify the situation and then build up problems carefully graded in difficulty until the one which caused trouble is reached."[5]

f. Individual Help. "The program for acquiring skills is being individualized, especially in respect to the amount of time given, the amount of repetition and practice provided, the nature of the materials learned, and the learning goals set up. Special individualized help is provided for those who learn slowly. Much more attention than formerly is given the slower child whose capacities and limitations are now so much better understood. We do not try to 'bring him up to normal' but rather to understand his needs and conserve his desirable assets, to start where he is, retain his interest and proceed in his instruction at the rate he can go.

"Modern teachers realize that within a given grade children may range several years in ability and achievement. Teachers meet this situation through varying requirements instead of attempting the impossible task of making all alike. Goals in skills are suited to the mental capacities of the learners and are differentiated in terms of learning abilities. No one expects every child to learn at the same rate as every other child, nor to attain a theoretical average."[6]

g. Practice. "Progressive education is confronted with the task of maintaining a free atmosphere on the one hand and systematizing the program to include provision for drill on the other. Systematic practice, it is well understood, can be very different from mechanical learning.

"Under normal conditions, practice *results* in acquiring new skills or in maintaining previously learned skills to a high degree of perfection. Practice does not necessarily make perfect, but the value of meaningful practice in improvement is demonstrated by many learning experiments as well as ordinary observations of learners during practice. If other conditions are favorable, repetition insures improvement.

"Practice appears to be effective in proportion to the desire to learn, the concentration during practice, meaningfulness of the activity practiced, the relation of the ability demanded in the skill to the learner's own ability, physical well-being of the learner, regularity of the practice, the duration and distribution of practice periods, and the nature of the skill being practiced.

[5] *Ibid.*, pp. 58 and 59.
[6] *Ibid.*, p. 13.

"Successful teaching or supervision of practice enters into the learning situation at this point. Uncontrolled, undirected, and unorganized practice may avail nothing. Directed selection of efficient learning efforts is more conducive to improvement in a child mature enough to profit from teaching."[7]

It is to be noted that some children will learn more than others because they differ in native curiosity, interest, and ability. "*Slower* learners require more drill, need more repetition, need more concrete illustrations, and more practice on separate facts than do the faster children. *Slower* pupils reason *less* accurately, use intuition *less* often in problem solving, *less readily* make adjustments to new features as they appear and do not concentrate as well."[8]

5. Examples of Practical Application

So far I have given you the best thinking of leading educators that I could find on this subject of correlating the arithmetic with the unit of study. How can these ideas of theory and method be put into practice? There must be several different ways. Every teacher has her own system. Briefly, I will explain the system I have used.

From the unit on *The Citrus Industry*, let me show you how concrete arithmetic needs arose. One of the first questions selected to work on by the class was: "How do citrus fruits keep us healthy?" The Health Committee was formed, and it began to do reference reading on the topic of vitamins. In the book *Health and Success* by Andress and Evans, Lois found a story about Cartier saving his explorers by feeding them a tea made from the leaves of an evergreen tree when their diet consisted of only dried meat or fish, and no fresh fruits and berries could be found. She made this report to the class. In the discussion that followed it was brought out that if Cartier lived now he would have fed his men canned tomato juice and citrus juice because recent research in health has taught people to have these juices in a balanced diet. "Well," asked Elisa, "When did Cartier live, anyway, not to know about balanced diets?" Lois did not know. Nobody knew. So Lois was delegated to find out. Thus did the following arithmetic problem arise: How long ago did Cartier live? What process are you going to use to solve this problem?

We live in 1940. Cartier lived in 1535. 1940 minus 1535 equals 405. Cartier lived 405 years ago. When Lois subtracted, she had a little difficulty in subtracting using a zero. Suitable practice was found in Book IV, on page 207, and Book V, on page 196.

Later on this same committee found a chart that gave the number of calories a person needs in his daily diet. In reporting this to the class, Greta said: "What are calories for?" Billy said: "I don't know yet." The next day, during the evaluation period, Greta said: "Billy, did you find out what calories are for?" Then Billy reported that an adult needs 3000 calories, a working man needs 5000 calories, a growing child between two and twelve years old needs 2500 calories. Then Lois asked: "Where do you get calories?" Billy answered, "From your food." Greta said: "Well, I don't understand." So Billy got the book and read aloud the section on calories to the class. Then everyone became interested in finding out if he was getting enough calories for his size. For a whole day each pupil recorded his diet, and added up the caloric value to see if it tallied with the chart. One child's paper read:

Breakfast		*Lunch*	
Milk	100 calories	Beef Soup	100 calories
Cereal	119 calories	Carrots	45 calories
Toast	100 calories	Sandwich	200 calories
Butter	77 calories	Milk	100 calories

Dinner

Milk	100 calories
Bread	100 calories
Butter	77 calories
Meat Stew	300 calories
Cake	300 calories
Salad (tomato)	50 calories

Total 1918 calories

This lesson gave practice in column adding, carrying, and comparison of numbers, and subtraction. Some pupils had diffi-

[7] *Ibid.*, pp. 18, 48, and 52.
[8] *Ibid.*, p. 185.

culty in the carrying, so suitable practice was found in Book IV on page 147.

Later on, the class became interested in the question of irrigation. The question was: Is irrigation as satisfactory as rainfall, in supplying water for citrus trees? This gave many arithmetic experiences. To answer this question the almanac was consulted to find that the annual rainfall for Southern California was between twelve and fifteen inches. Then it was found that for citrus trees, two and a half acre inches to four acre inches of water had to be applied monthly during the summer season. They then had to find out what an "acre inch" is. They decided that February was the month we usually had the most rainfall. A chart was kept of the rainfall in Santa Barbara for that month. This question gave the pupils the experience of writing, reading, and adding and multiplying decimals. Further practice in decimals was found on pages 103 and 104 in Book VI.

When the class took the excursion to the Goleta Packing House, the pupils learned that the women sorters made $2.67 per day. Later at school, this fact was mentioned, and questions were raised: Is that a lot of money for a worker to make? How much would that amount to for a week? A month? A season? How does that wage compare with that of the men workers? With that of other women doing different work? This problem brought up the necessity of multiplying with one and two digits, and using dollars and cents, as well as comparing figures again. These are a few ways in which arithmetic problems came up during the citrus unit.

Arithmetic Processes Needed to Carry on a Citrus Unit

Processes	*Lessons*
Addition	6
Subtraction	19
Short Division	12
Long Division	3
Multiplication	6
Large Numbers	6
Fractions	12
Decimals	3
Time	5
Scale	5
Cubic Measure	2
Percentage	1
Calendar	3
Money	1
Acres	4
Degrees	1
Measuring	14

Practices in Book V

Scope	*Pages*
Addition	9
Subtraction	9
Multiplication	13
Short Division	2
Long Division	25
Fractions	21
Measuring	3
Large Numbers	2
Degrees	3
Acres	1
Money	1

The citrus unit gave us many arithmetic situations and possibilities. This chart, "Arithmetic Processes Needed to Carry on a Citrus Unit," shows the wide variety of problems that arose requiring definite practice from the arithmetic textbook.

The next chart, "Practices in Book V," enables us to compare the practices in Book V; while the other chart lacks this concentrated effort, it makes up for this in two ways:

1. The wider variety of processes made arithmetic in the citrus unit much more interesting.
2. The processes in the citrus unit were a vital part of the activity and more meaningful, and, therefore, the class learned the processes in a shorter time.

How to Write the Problem. In simple vocabulary on the level of the reading ability of the class, write the story of the problem. Below the story ask suitable questions which will provide the solution to the problem when answered. Include in the questions one regarding the process or method used in solving it. Also include suitable practice material giving the pupils additional drill on the same process.

The practice usually is that in the arithmetic textbook.

A Truck Load

(A Sample Arithmetic Problem)

Tuesday we went to Mr. Hodges' Ranch. We saw a truck hauling little lemon trees to the orchard. The truck has 36 trees along the length and 11 trees along the width. How many trees were on the truck?

1. The truck carried——trees along the length.
2. There were——trees along the width.
3. How many trees were on the truck?
4. What process did you use to get this answer?
5. Practice: Book V, page 65.

Method of Solution.

1. Pass out to each pupil the arithmetic lesson on a mimeographed sheet.
2. The teacher gives suitable and appropriate remarks to call to mind the place and circumstances that the problem occurred.
3. *Expect* the class to make a conscientious effort to attempt to solve the problem without any help. At least each one should read it; answer some of the questions, etc. Allow adequate time for this work period.
4. Then at a designated time the class gathers as for any discussion or evaluation period. That is, those in the rear of the room occupy the chairs around the conference table.
5. Everyone, together, reads the story aloud. This gives the slower pupils another chance to get the whole story. Follow with any explanation necessary to make the story perfectly clear. Some pupil now reads the first question and calls on another for the answer. If there is disagreement, he must refer back to the story and read that part which will answer his question to the satisfaction of all. The rest of the questions are read and answered by other members of the class following the same plan. When there is a part no one can answer satisfactorily, a discussion follows, and by adroit questioning the teacher leads the pupils to understand the essential idea, using pictures, diagrams, or other meaningful symbols to represent the number aspect of the situation. She lets the pupils hear the language most potent to bring to his attention the arithmetic ideas in the problem, and finally helps him to generalize the arithmetic relationships in the problem.
6. Check here at the conference table, also, the practice examples. When all are not finished, get the pupils to realize that *here* is their opportunity to practice for their future needs. Suitable practice is not always found in the textbook designated for that particular grade. Very often we need to use the arithmetic book from another grade.

I have had teachers say to me that a unit on the study of the citrus industry had many arithmetic possibilities, but not in all units of study are they so plentiful. My answer is, if the unit is vital and meaningful to the children it will contain vital and meaningful arithmetic. And if the teacher is conscientiously *alert* for arithmetic problem situations, she will find more and more. She will become "arithmetic conscious" and so will her class. At all times the teacher needs to be alert and searching for the cropping up of arithmetic problems, especially during the discussion, work periods, and excursions. She can make a game of finding arithmetic problems in the unit with her pupils. Then there will be more problems to solve than time in which to do them. Then a finer selection can be made, and the more worth-while ones given attention to the exclusion of those too simple and easy. The arithmetic text need not be discarded but be made a very useful reference book along with the arithmetic books of all the grades below and at least the grade level above the one in which they are now. The use of the arithmetic book in the grade above as a reference for more practice will be a splendid challenge to the class and certainly won't be "making arithmetic harder than it is for the poor kiddies." Any teacher who conscientiously and actually tries this system I have outlined, will find, to her surprise and satisfaction, as I have, that it works—thus correlating arithmetic with the unit!

Teaching a Unit in Logical Reasoning in the Tenth Grade

By DANIEL B. LLOYD
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WE REALIZE the need for training our students to reason logically upon all questions that they may encounter, and we feel that all subjects in the curriculum should contribute to some extent to this end. But when and where in the curriculum should we best afford some systematically organized training in logical thinking?

To settle this, let us remember that we must choose a subject containing a cold body of facts about which we can argue and reason with unanimity and clear understanding as to basic assumptions; about which there is no diversity of opinion or varying shades of prejudice. Geometry furnishes more nearly that ideal content than does any other field. Therefore the principles and practice of reasoning can best be carried on in an organized way after the student has acquired the language of demonstrative geometry, usually in the tenth grade.

However, it seems to be too widely the practice in the teaching of demonstrative geometry to regard the ability to prove geometric theorems or originals as the ultimate aim in the course; and to this end certain techniques of reasoning and logic are inculcated with the hope that the student may be able to attack such proofs successfully. Even assuming that the student acquires this power, and it is admittedly worthy as far as it goes, it should really be considered a means to an end rather than an end in itself. Should it not be utilized to enable him to reason logically, and to recognize sound reasoning by others, in *all* life situations which he may later encounter? If we do not capitalize upon this opportunity here, it is lost and "gone with the wind."

Next, invoking the modern principles of transfer of training and the necessity of common elements therein, we are led inevitably to the necessity of providing the student with life reasoning situations. That is, there should be some training in non-geometric reasoning to assure carry-over.

No desire exists on the part of the author to establish the above thesis as an innovation as it has been already established in the literature. However, it was thought advisable to re-establish it here as a basis for a discussion of its implementation in an actual course of study.

To show that a thing should be done is *one* thing, whereas to reveal a suitable way of doing it is still *another*. The story is told of the butterfly and the chinch-bug, in which the bug, grovelling in the dirt all day long and envious of the butterfly flitting happily around in a world of sunshine, asked the latter how he too might fly freely and joyously hither and yon; whereupon the butterfly with the air of an educational philosopher responded impressively: "That is easy, my dear chinch-bug; you should merely change yourself into a butterfly." The chinch-bug became very happy at the thought of this simple solution, until, after contemplating it for a few minutes, the puzzling question came to his mind as to how he was to effect this transformation, and so asked an explanation of the butterfly. The latter replied: "I have merely attempted to lay down the general principles and expect you to work out the specific details."

In this paper the author will attempt a partial answer to the problem of practical implementation. He will discuss it by way of reference to some of the phases of the problem encountered in his own school system in Washington, D. C. Our city-

wide curriculum committees now attacking the problem of course of study content, are face to face with many such problems. In the tenth grade we are thinking of replacing some of the conventional plane geometry by units of general value—practical, disciplinary, and cultural, for those not requiring the geometry for college. Much along the line of consumer and community mathematics may be included, drawing upon arithmetic, algebra, and geometry to the extent needed to serve pragmatic and functional needs. But also to be stressed therein is the disciplinary aim. Consistent with this aim we feel that training in logical reasoning is essential. Although such training is incidental to all mathematical instruction, often what is most commonplace is least observed. At any rate a distinct unit in logical reasoning, such as is shown below, is being considered in this new course.

In deciding what concepts of logic are essential to include we believe there should be listed: the importance of definitions, the necessity of assumptions and some undefined terms, the way in which we combine a general statement and a specific statement to draw a conclusion; this leads naturally to the formal syllogism. A detailed study of the syllogism takes the sting of abstractness out of the meaning of geometric proof; and conversely, practice with many proofs facilitates deductive reasoning in general; they supplement and reinforce each other.

The unit on logical reasoning should go further than merely deductive reasoning. The inductive, or scientific method, should be developed and contrasted with the deductive, and widely illustrated from other fields.

The unit reaches its functional peak in the section dealing with errors in reasoning. We must expect that the illustrations here will be largely non-mathematical and we should not view this fact apologetically. We should be willing to scrap our mathematical lumber just as a builder removes the highly important scaffolding and falsework incident to the completion of any fine structure. Does not our mathematical symbolism of x, y, and z bloom forth into miles per hour, revolutions per minute, or foot-pounds of energy, as needed, leaving behind them no trace of their humble existence in the final product of man's mind? We will never weaken the tools of mathematics by putting them to still more potent uses. Using them to strengthen man's power to think is not desecration but rather glorification of the "Queen of the Sciences" and we should not sidestep such an opportunity to do so.

Unit on Logical Reasoning

A. Objectives of the Unit:
   1. Development of an understanding of the meaning and use of:
      a) Inductive and deductive reasoning.
      b) Postulational thinking and proof.
      c) Necessary and sufficient conditions.
      d) Indirect proof.
      e) Converse, inverse, and contrapositive.
      f) Syllogisms, valid and invalid (fallacies).
      g) Definition.
   2. Increased ability to reason logically in life situations.
   3. Increased ability to distinguish sound and unsound reasoning, such as in advertising and propaganda.

B. Content:
   1. Inductive and deductive methods; their uses in mathematics, science, and everyday life.
      Reference: N* pp. 46–50; F pp. 65–71.
      Generalizing; discovery of laws of nature; inductive-deductive technique in science; comparison with case method in study of law.
      Reference: N pp. 6–10, 14.
   2. Logical systems; postulates and axioms; Euclidean and non-Euclidean geometries as illustrations. Reference: Brown; N p. 30; THE MATH-

* Above references are made to the following bibliography; key to abbreviations is: N = Nyberg, "Exercises in Reasoning"; F = Fawcett, "The Nature of Proof."

EMATICS TEACHER. Feb. '40, pp. 73–79.

3. Implicit assumptions in non-mathematical reasoning; implications and inevitable consequences.
   Reference: F pp. 75–86; N pp. 3–6, 20; Ulmer.
   Necessary vs. sufficient conditions.
   Reference: N pp. 44–46, 1–2.
4. Indirect method of proof.
   Mathematical and non-mathematical illustrations.
   Reference: Upton; N pp. 34–37.
5. Converse, inverse, and contrapositive.
   Reference: Lazar; N pp. 34–37, 38–39, 40–43.
6. Definitions; their formation: genus and differentia.
   Reference: N pp. 18–19; F pp. 30–52.
7. Syllogisms; laws of the syllogism; abridged syllogism.
   Reference: N pp. 14–17; textbooks on logic.
   Euler circles.
   Reference: N pp. 67–69.
   Fallacies of the syllogism.
   Reference: N p. 52.
8. Mistakes in reasoning (with illustrations from current advertising or propaganda):
   - Circular reasoning (begging the question).
   - Denying the conclusion.
   - Hasty generalizations.
   - Misleading analogies.
   - Poor authorities.
   - Non-causal relations (non sequitur).
   - Confusion of necessary and sufficient conditions.
   - Assuming the truth of a converse or inverse.
   - Failure to exhaust all possibilities in an indirect argument.
   - Failure to distinguish between facts and assumptions.
   - Fallacies of diction.
     (Equivocation, amphibology, erroneous interpretation.)
   - Appealing to emotion or prejudice.
   - Dodging the issue.
     (Ad hominem arguments, pooh-poohing, etc.)

   Reference: N pp. 51–60.

BIBLIOGRAPHY

1. ATEN, HAROLD D. "A Reorganization of Geometry for Carryover," THE MATHEMATICS TEACHER, April 1941.
2. BLACK, M. "The Place of Logic in the Teaching of Geometry," *Mathematical Gazette*, Feb. 1939.
3. BRAVERMAN, B. "The Indirect Proof in a Life Situation," *High Points*, June 1937, pp. 77–78.
4. BROWN, C. H. "A Curriculum Unit on Non-Euclidean Geometry," *School Science and Mathematics*, Feb. 1940.
5. Charosh, Mannis. "Possible Transfer Values of 10th Year Mathematics," *High Points*, June 1941.
6. CHRISTOFFERSON, H. C. "Geometry a Way of Thinking," THE MATHEMATICS TEACHER, April 1938.
7. CHRISTOFFERSON, H. C. "Some Mathematical Concepts to Be Taught in the Tenth Grade," *School Science and Mathematics*, Jan. 1939.
8. CREIGHTON and SMART. *Introduction to Logic.*
9. DEMORGAN, A. *Formal Logic.*
10. FAWCETT, HAROLD P. "The Nature of Proof," National Council Teachers of Mathematics, 13th Yearbook, 1938.
11. HALL, E. L. "Applying Geometric Methods of Thinking to Life Situations," THE MATHEMATICS TEACHER, Dec. 1938.
12. HENDERSON, K. B. "Geometry Coup," *Clearing House*, April 1941, pp. 481–482.
13. LAZAR, NATHAN. *The Importance of Certain Concepts and Laws of Logic for the Study and Teaching of Geometry*, Doctor's Thesis, 1937, T. C., Columbia University. (Also published as articles in THE MATHEMATICS TEACHER, March, April and May 1938.)
14. NYBERG, JOSEPH A. *Exercises in Reasoning*, published by author, 50¢, Hyde Park High School, 10505 South Bell Avenue, Chicago. A useful workbook replete with exercises and illustrative teaching material.
15. NYBERG, JOSEPH A. "Teaching Geometry and Logic," *School Science and Mathematics*, March 1939.
16. ROBINSON, D. S. *The Art of Reasoning*, D. Appleton and Co., New York (1927). Typical of any good reference text on logic on the college level for teacher use; an introduction to logic and scientific method.
17. SHANHOLT, HENRY H. "A New Deal in Geometry," THE MATHEMATICS TEACHER, Feb. 1936.
18. ULMER, GILBERT. "Teaching Geometry for Logical Thinking," THE MATHEMATICS TEACHER, Dec. 1937.

19. UPTON, C. B. "The Use of Indirect Proof in Geometry and in Life," National Council of Teachers of Mathematics Fifth Yearbook, 1930, pp. 102–133.

No attempt has been made to make the above bibliography exhaustive, but rather to present a representative list of reading material from current literature sufficient to provide a teaching background. Several of the references should be available in the classroom for pupil as well as teacher reference. In fact copies of the booklet "Exercises in Reasoning," by Nyberg, could profitably be in the hands of each pupil during the study of the unit. Likewise, the work listed by Dr. Lazar, of Teachers' College, is a valuable piece of source material.

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The Function of Verbal Problems in Elementary Algebra

By FRED E. ELLIS
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TEACHERS of algebra have been on the defensive. Though a favorite subject of traditional educators, algebra has gradually been pushed further and further into the background of the secondary-school curriculum. The present emergency has given rise to a desperate need for mathematics. Must algebra flash across the educational sky only to flicker out when the emergency has passed? Unless teachers evaluate algebra in a more realistic light and take far-reaching steps to improve their instruction its now prominent position in the curriculum will be threatened when it ceases to be of immediate utility. It is not the purpose of this paper to examine the traditional methods of teaching but rather to appraise algebra as a technique for reflective thinking and to show the importance of verbal problems in training secondary-school children how to think reflectively.

The teaching of algebra has been dominated by a persistent and misplaced emphasis on manipulation. As E. T. Bell puts it:

> What shall it profit a mechanic or a surveyor if he can apply the rule of thumb formulas in his handbook as automatically as he breathes if he votes some plausible quack into office merely because he himself, in his practical, efficient blindness, is unable to distinguish between a sound argument and a tissue of rubbish.[1]

Granted that a certain skill in manipulation is necessary for understanding a mathematical argument, but it is pitifully inadequate for intelligent participation in modern society. With the exception of specialists technical skills are soon forgotten by most students. Regardless of manipulative skills high-school students should have a clear idea of the meaning of "proof" in deductive reasoning and realize that "proofs" are worth no more than the assumptions or postulates upon which they are based, or their training in algebra will have been futile.

All mathematics are systems of logic whereby conclusions may be deduced from hypothesized assumptions. Mathematics teaching must be grounded upon this definition and students must think clearly through its implications. If the basic assumptions of any argument can be shown to be false the conclusions must inevitably collapse. The following might well serve as objectives for teaching: (a) state all the postulates, (b) use only these postulates in the chain of reasoning, (c) remember that the most scintillating and attractive conclusion is no better than the assumptions upon which it is based. The teacher must impress upon the student the assumptive nature of mathematics, its rigidly abstract character and the basic fact that mathematics is not the science of number but a conceptual tool for describing and interpreting our environment.

In no other subject in the curriculum have teachers continued to worship the past than in mathematics, a truly deplorable situation in the light of the tremendous strides of mathematics since the beginning of the nineteenth century. The geometry of Lobatchewsky showed that mathematical "truths" once regarded as ultimate are but relative concepts of the human mind. Must we forever continue to preach to students about the inflexible "laws" of science and the eternal "verities" of mathematics? Mathematics and science have no laws and contain no

[1] E. T. Bell, "The Meaning of Mathematics," p. 138, *Mathematics in Modern Education, Eleventh Yearbook*, National Council of Teachers of Mathematics, 1936.

eternal verities. We must understand that the very bases of our philosophies of life, of our democratic society are not eternally fixed "laws" but are fluid and changing concepts. It is our task to decide through reason which of the premises we must refine in order to build a more perfect social structure. The very existence of society is threatened when it ceases to question the basis for its beliefs and actions and substitutes emotion for reason. To the process of reasoning and testing mathematics can be of the utmost value, in fact, a corollary of it.

We have assumed that a prerequisite to democratic society is the ability of each member to act on the basis of careful, reasoned argument. To what extent can the study of algebra stimulate and develop the reasoning powers of our students? All life is a succession of problems each of which is fraught with novel circumstances. We must adjust ourselves to these problems and bring to their solution the most powerful method at our disposal. This method is critical thinking. Insofar as mathematics helps us to think its study becomes necessary and its place in the curriculum is justified; if it merely teaches us to manipulate figures it should be rejected and something more useful put in its place. Mathematics makes its singularly valuable contribution to clear thinking through the verbal problem. It will be noted that the relationships between numerical quantities are not stated in verbal problems, whereas these relationships are plainly indicated in non-verbal exercises. The similarity of life problems to verbal problems in algebra is truly remarkable. Both require the selection and organization of relevant data; both are handled symbolically and the results of both must be checked in the situations from which they arose. It is true that students who are unable to handle tricks and puzzles become frustrated and develop a thorough hatred of mathematics. But why were those students forced to work such stupid problems; surely not because the mental anguish stimulated thinking! In the interests of students problems must be brought within the range of their common experiences. The judicious use of puzzles is justified only for the practice of setting up equations. It is not for the teacher to compel, but rather for the student to decide whether he wishes to accept the challenge of this type of problem.

Breslich lists the following characteristics which may serve as criteria for a good verbal problem:[2]

1. If it occurs frequently in life situations; i.e. in the activities of everyday life people must find it necessary to solve the problem.
2. If it seems feasible that it might arise in everyday life and have to be solved by someone.
3. If it occurs in the everyday activities of pupils, especially in school studies other than mathematics, e.g. science, mechanics, engineering, and mensuration.
4. If it is an individual problem that a particular pupil needs to solve.
5. If it has practical value.
6. If it has social value.

Problems about adult life-needs are often remote from the child's experience, hence practicality in itself is not always a suitable criterion of a good problem. Powell found that problems considered important by experienced teachers were not particularly interesting to students.[3] Only after the child has thoroughly explored and solved the problems of his world can he successfully undertake the solution of adult problems. Many "practical" problems are so obviously unreal as to be of slight value. How many of us recall struggling over "clock" problems and fantastic investment problems in which the investor had forgotten how much money he had put into an enterprise and had to compute it from the interest received! Students are quick to sense the

[2] E. R. Breslich, *Problems in Teaching Secondary School Mathematics*, p. 186, The University of Chicago Press, 1931.

[3] Jesse J. Powell, *A Study of Problem Material in High School Algebra*, p. 14. New York: Bureau of Publications, Columbia University, 1929.

ridiculous in such problems. Little wonder algebra is the target of scathing criticism.

The satisfactory solution of verbal problems will be determined largely by the student's ability to comprehend what he reads. Drake found a high correlation between reading ability and success or failure in solving problems.[4] Students trained in the vocabulary of algebra consistently scored higher than those to whom the instruction was not given. The implications of this study for the teaching of algebra are of major importance. In general children are not careful, analytical readers. Much of their training in reading has encouraged them to read rapidly for general ideas rather than to read slowly for details. They must be encouraged to seek out the implications of the verbal statements they read when adequate understanding of the material necessitates careful analysis. It is helpful for those students who consistently fail to note details in their reading to list the numerical quantities involved in a problem before attempting to solve it. Only after a thorough understanding of the relationships of these quantities should they proceed to symbolize the verbal statements and set up equations. An adequate solution of life problems likewise must develop from a thorough understanding of the nature and relationships of the factors involved; only then can thinking organize these data in terms of a purpo eful solution.

Problem solving in algebra might well be taught to students in terms of the following principles:

1. List all the numerical quantities.
2. Analyze the verbal statements and indicate the relationships between the numerical quantities.
3. Translate the verbal statements into the language of algebra.
4. Solve the symbolic statement or equation.
5. Check the result in the equation.

To summarize, we have found that the life problems of both children and adults are handled by essentially the same methods used to solve verbal problems in algebra. The primary purpose of education is to teach our youth to think; and insofar as mathematics, properly taught, will help to achieve that objective, its function will prove invaluable. Within the mathematics courses well-selected verbal problems will carry us far toward achieving that goal.

[4] R. M. Drake, "The Effect of Teaching the Vocabulary of Algebra," pp. 601–610, *Journal of Educational Research*, April 1940.

Selected Bibliography

Bell, Eric T., *The Search for Truth*, New York, Reynal and Hitchcock, 1934.

Butler, Charles H. and Wren, F. L., *The Teaching of Secondary Mathematics*, New York, McGraw-Hill Book Co., 1941.

Clifford, W. K., "The Ethics of Belief" in *Lectures and Essays*, New York, Macmillan Co., 1886.

Ligda, Paul, *The Teaching of Elementary Algebra*, Boston, Houghton, Mifflin Co., 1925.

McKim, Margaret Grace, *The Reading of Verbal Material in Ninth Grade Algebra*, Contributions to Education 850 (New York, Bureau of Publications, Teachers College, Columbia University, 1941).

Reeve, W. D. editor, "The Teaching of Algebra," *Seventh Yearbook*, National Council of Teachers of Mathematics, New York, 1932.

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The Graphical Solution of Mixture Problems

By Ralph Calvert
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Recently while discussing the graphical solution of certain problems with a class in college algebra, the idea occurred to me that mixture problems might be solved in this manner. The following discussion is meant to serve only as an introduction for those who have not met this type of solution before.

Perhaps the mixture problem most frequently met is that in which all data concerning the final mixture are known. The following problem will serve as an example:

A grocer mixes coffee worth 25¢ per pound with coffee worth 50¢ per pound. How much of each must he take to get ten pounds of coffee that will sell for 40¢ per pound?

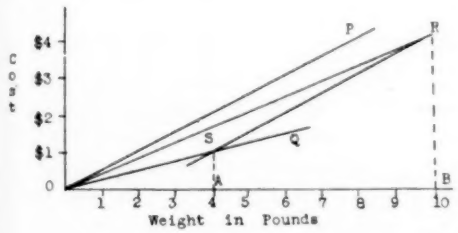

Fig. 1

In the solution of this problem (see Figure 1) plot pounds of coffee along the horizontal axis and total cost along the vertical axis with scales chosen so that the resulting lines have slopes that vary as much as is convenient. The lines OP and OQ, lengths not known, represent the relationships between cost and weight for the 50-cent coffee and the 25-cent coffee respectively. The line segment OR represents the final mixture of ten pounds at total cost of \$4. Through R construct $RS \| OP$ with S the point of intersection with OQ. Then OA represents the amount of 25-cent coffee and AB represents the amount of 50-cent coffee that must be added thereto. If ruled paper is used, RS may be drawn without the aid of OP.

Other mixture problems that may be solved by the same method as that discussed above follow:

1. The radiator of a car contains a mixture of alcohol and water that is 30% alcohol. How much must be drawn off and replaced with pure alcohol to give a mixture that is 55% alcohol?

2. One bar of metal is 35% copper and another is 18% copper. How much of each should be taken to make 50 oz. of metal that is 27% copper?

3. A flask contains 3 liters of acid and 8 liters of water while another flask contains 7 liters of acid and 11 liters of water. How much should be taken from each flask to make 9 liters of a mixture that is 35% acid?

Although the two problems that follow are not mixture problems in the ordinary sense, they will serve as examples of those that may be solved by the same methods:

1. A man invests \$5000, a part at 5% and the rest at 6%. If his income from both investments amounted to \$285 for the year, how much did he invest at each rate?

2. A certain project employs 130 men with a daily payroll of \$1030. If unskilled laborers earn \$6 per day and skilled laborers earn \$11 per day, how many of each work on the project?

Another common mixture problem is that in which all data concerning one of the initial mixtures are known. The three examples that follow are typical:

1. A dealer has 50 gal. of a 90-per cent alcohol solution. How much 40-per cent alcohol solution must be added to make a 70-per cent solution?

2. How many pounds of water must be evaporated from 100 pounds of a 5% salt solution to make a solution that is 8% salt?

3. A bar of metal weighing 75 pounds is 60% copper. How much pure copper must be melted with it to make a bar that is 85% copper?

The first of the preceding set of three problems is solved as indicated in Figure 2. Plot total gallons of solution along the horizontal axis and gallons of alcohol therein along the vertical axis. The line segment OP represents the 50 gal. of 90-per cent solution; ON represents the 40-per cent solution, amount unknown. Construct PQ parallel to ON (or with the

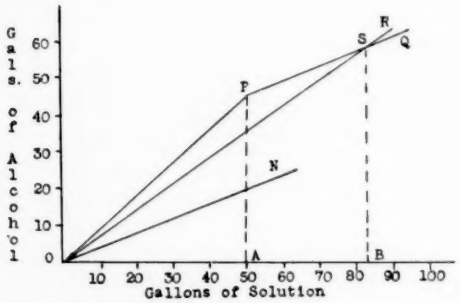

FIG. 2

proper slope, if ON is not drawn). Draw OR to represent a 70-per cent solution; OR and PQ intersect in the point S. The perpendiculars to the horizontal axis from P and S determine the points A and B respectively. Then AB gives the amount of 40-per cent solution added and OB gives the amount of the final solution.

A final example will serve to combine some of the ideas involved in the preceding problems. Consider the following problem in conjunction with Figure 3:

A container has in it a mixture of acid and water. If 50 qt. of acid are added, the ratio of acid to water is $\frac{7}{8}$. If 50 qt. of water are added, the ratio of acid to water is $\frac{1}{4}$. Find the strength of the solution.

In the solution of this problem plot the amount of mixture along the horizontal axis and the amount of acid therein along the vertical axis. A mixture that is 7/15 acid is represented by OM; one that is $\frac{1}{5}$ acid is represented by ON. Now we must find a point R such that RP, representing 50 qt. of added acid, and RQ, representing 50 qt. of added water, terminate on OM and ON respectively. (Pure acid adds equally to the amount of solution and to the amount of acid therein. Therefore RP is inclined 45° to the horizontal. Pure

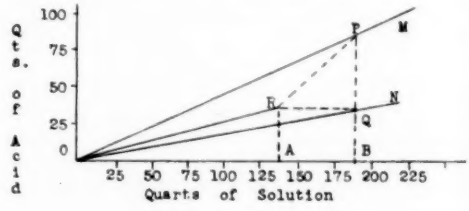

FIG. 3

water adds only to the amount of solution. Thus RQ is parallel to the horizontal axis.) Since $RQ = PQ$, the immediate problem is to find a place where the vertical distance between OM and ON represents 50 qt. On ruled paper this place may be estimated with sufficient accuracy and PQ, then RQ, drawn. The perpendicular from R to the horizontal axis determines the point A such that OA represents the amount of the original solution and RA represents the amount of acid therein.

Buy War Bonds and Stamps

Basic Mathematics

By RALPH MANSFIELD
Chicago Teachers College, Chicago, Illinois

DURING the past year, a continual stream of books has been appearing on the desks of many teachers of mathematics. Almost without exception, these books purport to expound "basic mathematics," to be taught to students who will one day become members of the armed forces of the United States.

It is bewildering, to say the least, to sort out from all these published works that which is basic and that which is non-basic; that which points and trains the students for military life and that which does not. For the most part, the appearance of these books can be attributed to a certain "war hysteria" which has taken hold of some few mercenary authors and brought them to the realization that there is now a "money market" for mathematical books. This "money market" takes advantage of the un-schooled persons—and in some cases, the schooled—who have had no previous training in mathematics and loads them up with books which they can neither comprehend nor need. And, even worse, it creates a demand for the type of mathematics that is tailor made for unhealthy consumption. It may be likened to the current craze for vitamin pills, where the persons who can afford the pills do not need them and those who need them cannot afford them.

There must be something fundamentally wrong in the teaching of mathematics when books such as these are not only eagerly purchased, but brought out in never ending procession. Putting the financial point of view aside, what is fundamentally wrong?

Perhaps the greatest fault lies in the fact that some mathematicians have failed to realize that all the training in mathematics required for the armed forces will be turned toward peacetime pursuits. Surely, no one can fail to admit that we must have meteorologists, communications technicians and engineers, transportation engineers, marine and aerial navigators, damage control engineers, and diesel engineers in our civilian life. The only change in these occupations for military adaptation lies in the sober fact that the persons performing these jobs wear uniforms and are more severely disciplined for their shortcomings in the performance of their duties.

Haven't we, as teachers of mathematics, been training people to go on into these fields in past years? Haven't we been aware of these opportunities in civilian life for our students? Or has the explosion of gunpowder startled us to a new wakefulness?

The majority of mathematics teachers have been wide awake to these things. But perhaps some of them feel that it is now necessary to substitute "projectile" for "point" in discussing parabolic motion and do similar things with their course subjects. Naturally, emphasizing particular applications in view of new terminologies is an admirable thing; but, fundamentally, we need to teach the same mathematics we have been teaching in civilian relationships in order to lead the neophyte inductee or selectee to a proper understanding of the basic concepts in mathematics courses. In other words, mathematics will not become more basic now than it has been by introducing a few pictures of airplanes, radio circuits, clouds and radios, as well as terminology that lies within the province of the military, into what might otherwise be our stock type of textbooks. Mathematics has always been a "basic subject" and always will be and wars will never make it any more "basic."

It is also true, however, that some mathematics teachers have failed, in pre-

vious years, to stress the importance of arithmetical skills—if they now realize this as a "basic" fault in their teachings and seek to remedy it, then the war will not have been fought in vain. This is an excellent opportunity to renew the fight that was once lost to the insistent demands of "socializing" forces to reduce the rigorous content of beginning courses in arithmetic.

Above all, though, mathematics teachers must keep clearly in mind that there will be a new post-war democracy, world wide in scope, coming out of this war. Greater planning for civilian life and the maintenance of the forces of democracy that annihilate the germs of fascist oppression will require large numbers of individuals trained in the basic science of mathematics. If we can stick by our guns and show, by convincing adherence to the fundamental concepts, that mathematics is a greater civilizing, educating, and democratizing force than the pseudobasic mathematics expounded in some of our present day literature, then mathematics will surely grow and flourish to its rightful magnitude in the post-war world.

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◆ THE ART OF TEACHING ◆

A New Technique in Handling the Congruence Theorems in Plane Geometry

By RALPH C. MILLER
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INTRODUCTION

THE USUAL method of proof employed in the congruence theorems kills, rather than stimulates, the interest of many students being introduced to geometry. The customary method of superposition applies some very nice axioms and postulates, but leaves the student mystified as to what it is all about. The fact the assumption, that an angle can be bisected, is used to prove the isosceles triangle theorem, which is used to prove the third congruence theorem (s.s.s. equals s.s.s.), which in turn is used to prove the original assumption (that an angle can be bisected) should contribute much to the added confusion of an alert student.

Probably most of the geometry teachers today try in some way to establish the proofs of the first two theorems (s.a.s. equals s.a.s. and a.s.a. equals a.s.a.) without looking too long at the detailed proofs in the text. The teachers are aware that superposition as used here frightens rather than interests the new student. But once over the hurdle these very teachers quickly take up the isosceles theorem as their first opportunity to carefully analyze a theorem and to impress the student with the form of the written proof. This theorem assumes the vertex angle can be bisected (but the teacher should not dwell too long at this point!). The third congruence theorem is now easily and interestingly proved by use of the isosceles theorem. This theorem is the basis for the proof of various elementary construction problems; one of these being the bisecting of an angle, which was assumed previously to prove the isosceles triangle theorem which was used to prove the s.s.s. theorem which we are now using to prove the construction of an angle bisector! "Reasoning in a circle" is one type of *thinking* we want students to recognize through the study of geometry. At this point in the textbook's presentation the teachers never call the attention of the class to this classic illustration of "circle reasoning."

Some other method of covering this introduction to the congruence theorems surely seems necessary by now. Even though the method may not be as rigorous as some mathematicians would desire, if the student is interested in what he is doing, sees where he is going and thoroughly enjoys it, the method is much better than our present one. We could then point out the beautiful piece of "reasoning in a circle" that is present in the textbook. (That is, if you didn't write the book!)

Postulating the three congruence theorems is a method used by many teachers to get their classes off to a good start. The students have learned three new sentences, but otherwise nothing else. I present here another method of introducing the first few theorems. I believe it not only is a logical presentation but one from which the student will begin to see the purpose of geometry.

Procedure

Using compass and straightedge, the student constructs a triangle using as sides the three sides of a *given* triangle he has drawn on his paper. The two triangles will certainly look congruent. By cutting out the triangles and placing one on the other, the student will accept their congruency without further proof. No, this doesn't hold to the best procedure in geometry, but the student now is convinced and satisfied with the statement that the triangles are congruent. Thus, the student will accept as a postulate—"Any two triangles are congruent if the three sides of one are equal to three sides of the other."

This method of superposition may not be using the best tenets of geometry, but at this point it is the most logical and desirable one. We are primarily concerned in getting a foundation upon which to build geometry in a logical manner.

Now, the student can construct an angle equal to a given angle by the usual method. The proof depends only on the congruence theorem above and the postulate that corresponding parts of congruent triangles are equal.

The remaining congruence theorems can be taken. Since the students can copy an angle, they can construct (not merely draw) a second triangle having s.a.s. equal s.a.s. of the original triangle. The third sides will be found to be equal by using only a compass for comparing lengths. Since the three sides are respectively equal in the two triangles, the triangles are congruent by s.s.s. equals s.s.s.

Let the student also copy a triangle by using a.s.a. equals a.s.a. The construction merely consists of drawing a line equal to a given side and constructing angles on both ends equal to the corresponding angles of the given triangle. Again by measuring the respective sides they will be found to be equal and the triangles congruent. Now the second and third theorems are proved to the student's satisfaction, but we can have the theorems accepted as postulates to keep from offending the mathematicians—"Any two triangles are congruent if two sides and including angle of one are equal to the two sides and including angle of the other," and "Any two triangles are congruent if two angles and the including side of one are equal to two angles and the including side of the other."

Of course, we are using the direct measurement of the lines as proof that they are equal. Although we don't resort to this method in the future, it will be more convincing to the student than the usual textbook methods.

The student now can construct the bisector of an angle and *prove it* is the bisector. Further, he can construct the perpendicular bisector of a line, and *prove* the construction. *He now has not one but two methods* of attack on the proof of the isosceles triangle theorem. He may (a) actually bisect the vertex angle and proceed as usual; or (b) he may bisect the base and connect this point with the vertex and prove the triangles are congruent to get the base angles equal.

The isosceles triangle theorem now has provided a real challenge to a class as two *real* and *interesting* methods of proof appear. The students alone will find both methods. This theorem isn't the beginning of the "circle reasoning," but the beginning of real analytical thinking.

Advantages

Although this method of handling the introduction to geometry may lack the rigor some mathematicians so desire, it does not lack rigor in the broader sense. The students have geometry presented here in the most logical form. They are not *drawing* angles, triangles, and angle bisectors as is done by the usual textbook presentation; they are *constructing* angles, triangles, and angle bisectors. I contend that, on one hand the method here is equally as rigorous as the general method, and on the other possesses several outstanding advantages.

1. *All* the students will understand what is going on.
2. They will be convinced of the congruency of the triangles.
3. The students will construct triangles having s.a.s. and a.s.a. equal respectively to a given triangle. The triangles will look congruent because of the accurate drawings.
4. The "reasoning in a circle" fallacy will be omitted. The isosceles triangle theorem comes afterward, not previous to the s.s.s. theorem.
5. In three days the students have hurdled the obstacles and pitfalls of the usual introduction to geometry. Furthermore, they are interested in the work.

Summary

This introduction through the congruence theorems is briefly stated below. It is important to keep in mind that all lines and figures are constructed (not simply drawn) from the first theorem.

THEOREM I: *Two triangles are congruent if three sides of one are equal to three sides of the other.*

POSTULATE: *Corresponding parts of congruent triangles are equal.*

CONSTRUCTION I: *To reproduce a given angle.*

THEOREM II: *Two triangles are congruent if two sides and the including angle of one are equal to the two sides and the including angle of the other.*

THEOREM III: *Two triangles are congruent if two angles and the including side of one are equal to the two angles and the including side of the other.*

CONSTRUCTION II: *To bisect a given angle.*

CONSTRUCTION III: *To bisect a given line.*

THEOREM IV: *The base angles of an isosceles triangle are equal.*

(*a*) *Prove by bisecting vertex angle.*

(*b*) *Prove by bisecting base.*

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The American Mathematical Monthly

February, 1943, vol. 50, no. 2.

1. Haurwitz, B., "The Application of Mathematics in Meteorology," pp. 77–84.
2. Keller, M. W., Shreve, D. R., and Remmers, H. H., "Diagnostic Testing Program in Purdue University," (Part 3. A Report of the Results of the Program), pp. 85–90.
3. Stabler, E. R., "Boolean Algebra as an Introduction to Postulational Methods," pp. 106–110.
4. Yates, R. C., "Classification of the Conics," pp. 112–115.
5. Hohn, F. E., "An Existence Proof for Logarithms," pp. 115–116.

Bulletin of the Kansas Association of Teachers of Mathematics

October, 1942, vol. 17, no. 1.

1. "Report of Kansas Mathematics Test Number Three in Arithmetic and Algebra," pp. 3–6.
2. Edwards, Lon, "Mathematics and Youth," pp. 6–8.
3. Holroyd, Ina E., "Some Problems that Cast Long Shadows into the Future," pp. 8–12.
4. Mallory, A. E., "The Pedagogical Significance of Plane Geometry," pp. 12–16.

National Mathematics Magazine

January, 1943, vol. 17, no. 4.

1. Sanders, S. T., "Leveller of Minds," p. 146.
2. Yanosik, George A., Graphical Solutions for Complex Roots of Quadratics, Cubics, and Quartics, pp. 147–150.
3. Barnhart, C. A., "Geometric Examples of Convergent Series," pp. 159–162.
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February, 1943, vol. 17, no. 5.

1. Sanders, S. T., "Mathematical By-Products," p. 194.
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3. Miller, O. A., "A Fifth Lesson in the History of Mathematics," pp. 212–220.
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School Science and Mathematics

March, 1943, vol. 43, no. 3.

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4. Christofferson, H. C., "Mathematics Basic to Navigation," pp. 254–264.
5. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 264–267.

Scripta Mathematica

September 1941, vol. 8, no. 3.

1. Karapetoff, Vladimir, "A General Outline of Restricted Relativity," pp. 145–163.
2. Curiosa, p. 164.
3. Simons, Lao Genevra, "Undergrauate Publications in Mathematics," pp. 165–175.
4. McCoy, John Calvin, "The Anatomy of Magic Squares," pp. 183–187.
5. Cohen, Joseph George, "Geometrical Ideas in Grammar," p. 188.

December, 1941, vol. 8, no. 4.

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